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Numerical methods and parallel algorithms for computation of periodic responses of plates

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ABSTRACT

Numerical methods that compute the nonlinear frequency–response curves of plates are presented. The equation of motion of the plate is derived by assuming first order shear deformation theory and considering geometrical type of nonlinearity. It is discretized by the finite element method using bilinear quadrilateral elements. The frequency–response curve is computed by shooting and continuation methods. Parallel implementation of the shooting method is presented and its scalability is investigated.

1. Introduction

Plates are basic structural components which have variety of applications in different branches of industry. Often these structures have to operate under extreme load conditions which lead to displacements with large amplitudes. The linear models are not appropriate to predict sufficiently accurate time response of the plate in the presence of large displacements, thus one should consider nonlinear theories. On the other hand, the nonlinear models behave very differently from the linear models, they can have more than one solution, bifurcation points, quasi-periodic or chaotic motions can also exist.

The response of nonlinear system can change significantly due to small changes of some of the parameters [1]. Thus, the parametric study of nonlinear dynamical systems is essential and important in the design, maintenance and health monitoring of elastic structures. Common tools for performing dynamical analysis of elastic structures are the nonlinear normal modes, for free vibration problems, and the nonlinear frequency–response curves also known as nonlinear frequency–response function, for forced vibration problems [2]. In both cases, the solution is considered to be periodic.

Nonlinear vibrations of plates have been studied by many researchers in the last decades. Mostly rectangular and circular plates were analyzed, not only because of their wide applications, but also because these plates can be discretized in space with few degrees of freedom. Hierarchical basis of shape functions is used in [3] for discretization of the equation of motion of rectangular plate. The nonlinear dynamical analysis is performed by expressing the vector of generalized coordinates in Fourier series and solving the resulting algebraic system by continuation method. Nonlinear normal modes are computed and it is shown that there is interaction between the modes due to internal resonances. Nonlinear vibrations of rectangular plates with different boundary conditions are investigated in [4]. The bifurcation analysis is performed by the software AUTO [5] which is developed for continuation and bifurcation analysis of nonlinear ordinary differential equations. Rectangular laminated plates are analyzed in [6]. The nonlinear equation of motion is discretized by using the natural modes of vibration which are obtained preliminary by solving the linear eigenvalue problem by expanding the displacements in terms of Chebyshev polynomials. The dynamical analysis is performed by using the collocation and the continuation methods and the nonlinear frequency–response curves are presented. Shooting method is used to find periodic responses

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of plates composed of laminates of variable stiffness [7]. The equation of motion is derived by assuming third order shear deformation theory and discretized in space by hierarchical basis of shape functions. It is shown in [8] that even harmonics appear in the periodic response of the plate even when the nonlinearity is only cubic. The appearance of even harmonics is due to a bifurcation point.

The nonlinear vibrations of circular plates are investigated in [9,10]. The study of axisymmetric vibrations of circular plates allows one to reduce the equation of motion into one dimensional equation in space along the radius of the plate. Axisymmetric responses of circular plates due to harmonic excitations are investigated by the method of multiple scales [9]. Coupling of the modes due to internal resonances are shown. Axisymmetric vibrations of circular plates are investigated by the harmonic balance method in [11]. Asymmetric nonlinear vibration of circular plates are studied in [12] by expressing the deflection as a function of the linear modes. Hierarchical set of shape functions is used in [10] to investigate the asymmetric vibration of circular plates. Interaction between symmetric and asymmetric modes of vibration is shown due to internal resonances.

In real applications, plates have complex geometries and space discretization requires the usage of the finite element method. Often, due to accuracy requirement, a fine mesh of elements is used for discretization which leads to large-scale systems. The computations of the nonlinear normal modes and nonlinear frequency response curves of large-scale nonlinear systems become computationally expensive and require the usage of powerful supercomputers and parallel algorithms.

In the current work, numerical methods for computing the nonlinear frequency–response curves of large-scale nonlinear systems are presented. The periodic responses are computed by shooting method and continuation method is used for defining prediction for next point from the frequency–response curve. Plate structures are analyzed and the equation of motion of the plate is derived by considering Mindlin's theory, also known as first order shear deformation theory [13]. Space discretization is achieved by the finite element method and using bilinear quadrilateral elements [14]. These elements allow one to apply the finite element method to plates with complex geometries. Reduced and selective integration is performed to avoid shear locking [15]. Currently, only transverse displacements of the plate are considered, because the main aim of the paper is to investigate the scalability of the shooting method applied to nonlinear large-scale dynamical systems and to present the potential of the proposed parallel implementation of the numerical method to investigate the dynamics of real-life complex structures. The future developments will consider in-plane displacements and application of the methods to curved structures, discretized by curved finite elements.

The complete computation of the nonlinear frequency–response curve is iterative process which involves matrix–matrix products and solutions of sparse and dense systems. When the initial system is large, the whole process becomes time consuming. Parallel implementation of the shooting method, together with parallel computation of the nonlinear stiffness matrix and the Jacobian are presented and the efficiency of the algorithm is investigated. The appropriate parallel implementation of the algorithm is essential for the application of the method to real-life structures. Finally, the frequency–response curve of plate with complex geometry is computed and presented.

2. Mathematical model

A plate with uniform thickness h and arbitrary geometry is assumed. The displacement components along the coordinate axes x, y and z are denoted by u, v and w, where u and v denote in-plane displacements and w represents the transverse (or our-of-plane) displacement of the plate. The displacements depend on space coordinates and time t. Following Mindlin's formulation for thick plates [16], the displacement components are expressed by the transverse displacement of the middle plane and the rotations of the middle plane about x and y axes:

$$\begin{aligned} u\,(x,y,z,t) &= z\phi_y(x,y,t),\\ v\,(x,y,z,t) &= -z\phi_x\,(x,y,t)\,,\\ w\,(x,y,z,t) &= w_0\,(x,y,t)\,, \end{aligned} \tag{1}$$

where w_0 represents the transverse displacement of the middle plane, ϕ_x and ϕ_y represent the rotations of the middle plane about x and y axes, respectively. The middle plane is defined for $z = 0$.

Geometrical type of nonlinearity is considered in the model, i.e. nonlinear strain–displacement relations are used. The von Kármán nonlinear strains associated with the displacements from Eq. (1) are given by ($\varepsilon_z = 0$):

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = z\frac{\partial \phi_y}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2,\\ \varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = -z\frac{\partial \phi_x}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2,\\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = z\frac{\partial \phi_y}{\partial y} - z\frac{\partial \phi_x}{\partial x} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y},\\ \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = -\phi_x + \frac{\partial w_0}{\partial y},\\ \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = \phi_y + \frac{\partial w_0}{\partial x}. \end{aligned} \tag{2}$$