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Updating a progic ☆

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ABSTRACT

This paper presents a progic, or probabilistic logic, in the sense of Haenni et al. [8]. The progic presented here is based on Bayesianism, as the progic discussed by Williamson [15]. However, the underlying generalised Bayesianism differs from the objective Bayesianism used by Williamson, in the calibration norm, and the liberalisation and interpretation of the reference probability in the norm of equivocation. As a consequence, the updating dynamics of both Bayesianisms differ essentially. Whereas objective Bayesianism is based on a probabilistic re-evaluation, orthodox Bayesianism is based on a probabilistic revision. I formulate a generalised and iterable orthodox Bayesian revision dynamics. This allows to define an *updating* procedure for the generalised Bayesian progic. The paper compares the generalised Bayesian progic and Williamson's objective Bayesian progic in strength, update dynamics and with respect to language (in)sensitivity.

0. Introduction

The question of combining logic with probability is here addressed within the framework of a *progic* in the sense of Haenni et al. [8]. Whereas Carnap combined logic and probability such that logic imposes constraints on probability in the form of invariance axioms, a progic is a logic for inferring probabilistic statements from accepted probabilistic premisses. Whereas Carnap transferred logical constraints onto epistemic probability to obtain his continuum of inductive probabilistic logics, a progic is rather an epistemic theory of probabilistic acceptance sharing similarities with the theory of belief revision (in a broad sense).

Section 1 introduces Williamson's objective (W-)Bayesianism and the generalised (G-)Bayesianism adopted here, and compares them. I show that W- and G-Bayesianism differ in at least four fundamental aspects: in the epistemic dynamics (re-evaluation vs. proper update), in the calibration norm (convex

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hull simpliciter vs. convex hull of the topological closure), in form (disjunctive vs. non-disjunctive) and finally because G-Bayesianism satisfies a principle of no divergence – negligible input differences should not make a difference in acceptance – which W-Bayesianism violates.

Section 2 introduces a more general presentation of progics in the sense of Haenni et al. [8]. I argue that this generalisation increases the expressive power (Section 2.1). In particular, non-simple linear constraints (that is, linear constraints with non-$0,1$ coefficients) are now expressible and certain natural uncountable infinite disjunctions reduce to conjunctions of two linear inequality constraints. I will then introduce the standard progic (Section 2.2), induced by logic and the axioms of probability, and argue that according to this inference relation, the premisses are to the conclusion as a belief core is to the propositions believed. Finally, I define the progics for W- and for G-Bayesianism in the case of consistent premisses (Section 2.3) and state the relation between these progics and the standard progic in terms of strength. In particular, I show that the W- and the G-progic are (strictly) proper inference relations (most of the time) or track probabilistic deduction: if J can be deduced from I – according to the standard progic – then J can be inferred from I – according to the Bayesian progics – (but not vice versa). This non-invertible relation is due to an epistemic dependence built into the progics. In certain cases the two progics are not proper. Here the epistemic dependence of the progic does not introduce preference bias, but rather indecision or caution.

Section 3 discusses different treatments in the case of inconsistent premisses or constraints. Section 3.1 discusses the option of taking the intersection of the set of maximal consistent subsets of the premisses [15]. This option is order-independent and requires an agent to always keep track of what she has already accepted. Section 3.2 contrasts this option with (belief or rank) revisions of the previously accepted constraints by a newly accepted constraint. I reject the latter, because introducing these new structures may also introduce similar (or new) problems which do not have a ready solution (e.g. the iteration problem for revision, or the problem of lowering an infinite rank). For this reason, I develop (Section 3.3) a probabilistic revision of the inference relation based on Minimising the Kullback–Leibler divergence (MinKL). Section 3.4 completes this approach by suggesting two routes for dealing with the case where MinKL is undefined, namely when the new constraint necessitates raising some probabilistically neglected possibilities. The first route comes down to first extending a probability by a convex combination with the equivocator on the newly suggested possibilities, and only then updating. The second route generalises the Kullback–Leibler divergence such that its default value for Q not absolutely continuous w.r.t. P has a non-standard value higher than any real number (and does not simply diverge to infinity). The two options coincide under certain conditions, which I take to be a convergence argument for these options.

Finally, in Section 4 I briefly discuss the language-(in)sensitivity of the progic in analogy to Williamson [15]. I show that the G-progic (as the W-progic) is language-insensitive if the equivocator is taken as reference probability. This result carries over (under a slight modification of its meaning) to an arbitrary reference probability, provided the reference probability is taken into the “equation” of the comparison. I then argue that the fact that the G-progic also satisfies the principle of indifference is not harmful and only means that, as expected, this progic does not distinguish what is indiscernible according to the premisses.

Section 5 summarises the discussion and the Appendices A–D contain the proofs of the main results.

1. Generalised vs. objective Bayesianism

In this section I introduce the *generalised* (G-)Bayesianism adopted here and compare it to the *objective* (W-)Bayesianism defended by Williamson [14,15], first informally (Section 1.1), then more formally (Sections 1.2–1.5). Depending on the interpretation of “reference probability”, G-Bayesianism may either be interpreted as generalised objective Bayesianism or as generalised orthodox Bayesianism (i.e. a generalisation of conditionalisation and Jeffrey conditionalisation). I will tend to interpret it as generalised orthodox Bayesianism. This is the *difference in epistemology* between the two Bayesianisms. A second difference, one of *form*, consists in the fact that, whereas G-Bayesianism reduces to a single norm on credences (the