

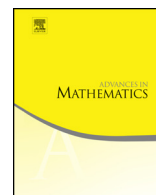


ELSEVIER

Contents lists available at ScienceDirect

Advances in Mathematics

www.elsevier.com/locate/aim



CrossMark

On conjugacy of Cartan subalgebras in extended affine Lie algebras

V. Chernousov^{a,*}, E. Neher^{b,2}, A. Pianzola^{a,c,3}, U. Yahorau^{a,4}

^a Department of Mathematics, University of Alberta, Edmonton, Alberta T6G 2G1, Canada

^b Department of Mathematics and Statistics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada

^c Centro de Altos Estudios en Ciencia Exactas, Avenida de Mayo 866, (1084) Buenos Aires, Argentina

ARTICLE INFO

Article history:

Received 17 September 2014

Received in revised form 20

November 2015

Accepted 28 November 2015

Available online 29 December 2015

Communicated by Roman

Bezrukavnikov

Keywords:

Extended affine Lie algebra

Lie torus

Conjugacy

Cartan subalgebra

Reductive group scheme

Non-abelian cohomology

ABSTRACT

That finite-dimensional simple Lie algebras over the complex numbers can be classified by means of purely combinatorial and geometric objects such as Coxeter–Dynkin diagrams and indecomposable irreducible root systems, is arguably one of the most elegant results in mathematics. The definition of the root system is done by fixing a Cartan subalgebra of the given Lie algebra. The remarkable fact is that (up to isomorphism) this construction is independent of the choice of the Cartan subalgebra. The modern way of establishing this fact is by showing that all Cartan subalgebras are conjugate.

For symmetrizable Kac–Moody Lie algebras, with the appropriate definition of Cartan subalgebra, conjugacy has been established by Peterson and Kac. An immediate consequence of this result is that the root systems and generalized Cartan matrices are invariants of the Kac–Moody Lie algebras. The purpose of this paper is to establish conjugacy of Cartan subalgebras for extended affine Lie algebras; a natural class

* Corresponding author.

E-mail addresses: chernous@math.ualberta.ca (V. Chernousov), neher@uottawa.ca (E. Neher), a.pianzola@math.ualberta.ca (A. Pianzola), yahorau@ualberta.ca (U. Yahorau).

¹ V. Chernousov was partially supported by the Canada Research Chairs Program and an NSERC research grant.

² E. Neher was partially supported by a Discovery grant from NSERC (008836-2011).

³ A. Pianzola wishes to thank NSERC and CONICET for their continuous support.

⁴ Present address: Department of Mathematics and Statistics, University of Ottawa, Ottawa, Ontario K1N 6N5, Canada.

of Lie algebras that generalizes the finite-dimensional simple Lie algebra and affine Kac–Moody Lie algebras.

© 2015 Elsevier Inc. All rights reserved.

Introduction

Let \mathfrak{g} be a finite-dimensional split simple Lie algebra over a field k of characteristic 0, and let \mathbf{G} be the simply connected Chevalley–Demazure algebraic group associated to \mathfrak{g} . Chevalley’s theorem [11, VIII, §3.3, Cor de la Prop. 10] asserts that all split Cartan subalgebras \mathfrak{h} of \mathfrak{g} are conjugate under the adjoint action of $\mathbf{G}(k)$ on \mathfrak{g} . This is one of the central results of classical Lie theory. One of its immediate consequences is that the corresponding root system is an invariant of the Lie algebra (i.e., it does not depend on the choice of Cartan subalgebra).

We now look at the analogous question in the infinite dimensional setup as it relates to extended affine Lie algebras (EALAs for short). We assume henceforth that k is algebraically closed, but the reader should keep in mind that our results are more akin to the setting of Chevalley’s theorem for general k than to conjugacy of Cartan subalgebras in finite-dimensional simple Lie algebras over algebraically closed fields. The role of $(\mathfrak{g}, \mathfrak{h})$ is now played by a pair (E, H) consisting of a Lie algebra E and a “Cartan subalgebra” H . There are other Cartan subalgebras, and the question is whether they are conjugate and, if so, under the action of which group.

The first example is that of untwisted affine Kac–Moody Lie algebras. Let $R = k[t^{\pm 1}]$. Then

$$(0.0.1) \quad E = \mathfrak{g} \otimes_k R \oplus kc \oplus kd$$

and

$$(0.0.2) \quad H = \mathfrak{h} \otimes 1 \oplus kc \oplus kd.$$

The relevant information is as follows. The k -Lie algebra $\mathfrak{g} \otimes_k R \oplus kc$ is a central extension (in fact the universal central extension) of the k -Lie algebra $\mathfrak{g} \otimes_k R$. The derivation d of $\mathfrak{g} \otimes_k R$ corresponds to the degree derivation td/dt acting on R . Finally \mathfrak{h} is a fixed Cartan subalgebra of \mathfrak{g} . The nature of H is that it is abelian, it acts k -diagonalizably on E , and it is maximal with respect to these properties. Correspondingly, these algebras are called MADs (Maximal Abelian Diagonalizable) subalgebras. A celebrated theorem of Peterson and Kac [24] states that all MADs of E are conjugate (under the action of a group that they construct which is the analogue of the simply connected group in the

Download English Version:

<https://daneshyari.com/en/article/6425308>

Download Persian Version:

<https://daneshyari.com/article/6425308>

[Daneshyari.com](https://daneshyari.com)