Cold Regions Science and Technology 119 (2015) 75–83

Contents lists available at ScienceDirect

Cold Regions Science and Technology

journal homepage: www.elsevier.com/locate/coldregions

A Model for the Electrical Resistivity of Frozen Soils and an Experimental Verification of the Model

Wei Shan *, Yao Liu, Zhaoguang Hu, Jitao Xiao

Institute of Cold Regions Science and Engineering, Northeast Forestry University, Harbin 150040, China

ARTICLE INFO

Article history:
Received 8 October 2014
Received in revised form 16 July 2015
Accepted 23 July 2015
Available online 12 August 2015

Keywords:
Unsaturated cohesive soil
Frozen soil
Electrical resistivity
Electrical resistivity model

ABSTRACT

A fraction of the pore water of a soil body undergoes a phase change during the freezing process. Therefore, the electrical resistivity properties of frozen soils are different from those of unfrozen soils. To thoroughly investigate the conductive properties of frozen soils, a theoretical model for the electrical resistivity of frozen soils was deduced and established, and the factors that affect the electrical resistivity of frozen soils were analyzed. Through experiments performed on frozen clay, the characteristics of the effects of the unfrozen water content, initial water content, soil temperature and dry density on the electrical resistivity of a frozen soil were analyzed. The model for the electrical resistivity of frozen soils indicates that the electrical resistivity exhibits a temperature-dependent inverse proportionality with the unfrozen water content of the soil body; the electrical resistivity of a frozen soil exhibits a complex temperature-related functional relationship with the initial water content; the electrical resistivity of a frozen soil is exponentially related to the soil temperature; and the electrical resistivity of a frozen soil is inversely proportional to the dry density of the soil body. The results of the experiments performed on frozen clay verify the reasonableness of the proposed model for the electrical resistivity of frozen soils.

1. Introduction

Electrical resistivity is an innate attribute of a soil. The magnitude of the electrical resistivity of a soil is determined by the dry density, water content, temperature, mineral composition and structure of the soil (Abu-Hassanein et al., 1996; Archie, 1942; McCarter and Desmazes, 1997). A previous experimental study demonstrated that the cementation factor is related to the shapes and cementation conditions of soil particles (Wyllie and Gregory, 1953). Wu et al. (1985) conducted a preliminary study of the factors that affected the variation in the electrical resistivity of soils under in-situ conditions; they found that the electrical resistivity of the soil varied with the soil type, mother rocks, soil texture and soil salt content. The electrical resistivity of a cement–soil mixture is well correlated with its water content, water–cement ratio and degree of saturation (Liu et al., 2006). Li et al. (2012) conducted a study of the relationship between the electrical resistivity of a saline soil and the water content, salt content, porosity and degree of saturation. They observed that the electrical resistivity of the saline soil decreased with the increasing water content, salt content and degree of saturation, and it increased with the increasing porosity. Zha et al. (2013) performed a laboratory study of the effect of the particle composition of a soil on the electrical resistivity of the soil and found that the electrical resistivity of the soil decreased with the increasing liquid limit or plastic limit of the soil.

Fortier et al. (1994) conducted a calorimetric experiment on an undisturbed frozen soil sample and obtained the unfrozen water and ice contents in the sample; in addition, Fortier et al. (2008) also determined the electrical resistivity at the location near the sampling site and obtained the relation between the electrical resistivity and the unfrozen water and ice contents through linear regression. Delaney et al. (2001) conducted a study of the electrical resistivity of frozen and petroleum-polluted soils; they found that the freezing conditions and petroleum pollution both could result in an increase in the electrical resistivity of the soil. Fu et al. (2009) monitored the electrical resistivity of silty clay that was obtained from the Beiluhe River on the Qinghai–Tibet Plateau during an entire uniaxial compression test at different temperatures. They found that the uniaxial compression strength of the frozen soil exhibited a strong semi-logarithmic relationship with the initial electrical resistivity. Angelopoulos et al. (2010) investigated frozen soil from Parsons Lake in the Northwest Territories of Canada using the electrical resistivity method and obtained the relationship between the electrical resistivity and ice content of the frozen soil. The electrical resistivity method was used to investigate the spatial distribution of the island-shaped permafrost layer along the Beian–Heihe highway (Hu and Shan, 2011). They observed discontinuities in the electrical resistivity at the upper and lower interfaces of the island-shaped permafrost layer, and the permafrost layer exhibited significantly high resistance. In areas without permafrost, the variation in the electrical resistivity was relatively gentle, and no discontinuity was observed.

To thoroughly study the electrical conductive properties of frozen soils, the present study investigates the relationship between the

---

* Corresponding author. Tel.: +86 451 82191477; fax: +86 451 82191590.
E-mail address: shanwei456@163.com (W. Shan).

http://dx.doi.org/10.1016/j.coldregions.2015.07.010
0165-232X/© 2015 Elsevier B.V. All rights reserved.

electrical resistivity of a soil body and the water content, temperature and dry density of the soil body by establishing a model for the electrical conductivity of frozen soils, using mathematical deduction and a theoretical model for the electrical resistivity of frozen soils. Moreover, experiments on soil bodies with different water contents and dry densities at different temperatures are conducted. The present study also verifies the reasonableness of the theoretical model for the electrical resistivity of frozen soils and provides a theoretical basis for exploring the distribution of underground shallow frozen soils using the electrical resistivity method.

2. Establishing a model for the electrical resistivity of frozen soils

2.1. Models for the electrical resistivity of soils

An electrical resistivity model that is applicable to saturated non-cohesive soils and pure sandstones, assuming that the conductivity of solid particles is not considered, has been proposed (Archie, 1942):

$$\rho = a\rho_w n^{-m} \tag{1}$$

where ρ is the electrical resistivity, ρ_w is the electrical resistivity of pore water, n is the porosity, a is an experimental parameter, and m is the cementation factor.

The electrical resistivity model proposed by Archie (1942) relates the electrical resistivity of a soil to the structure of the soil and expands approaches for studying the microstructures of soils. However, the electrical resistivity model proposed by Archie (1942) only considers the effect of the electrical resistivity and porosity of the pore water on the electrical resistivity of the soil; therefore, the potential applications of the electrical resistivity model proposed by Archie (1942) are limited.

In later work, the electrical resistivity model proposed by Archie (1942) was expanded to the following:

$$\rho = a\rho_w n^{-m} {s_r}^{-p} \tag{2}$$

where s_r is the degree of saturation and p is the saturation exponent.

The expanded electrical resistivity model considers the degree of saturation of the pore water; therefore, the expanded model is applicable to non-saturated pure sandstones and non-cohesive sand. However, the expanded model ignores the effects of other factors on the electrical resistivity of a soil.

Considering the effect of the electrical double layers on the surfaces of soil particles on the electrical resistivity of the entire soil body, and on the basis of experimental studies, an electrical resistivity model that is applicable to non-saturated cohesive soils was proposed (Waxman and Smits, 1968):

$$\rho = \frac{a\rho_w n^{-m} {s_r}^{1-p}}{s_r + \rho_w BQ} \tag{3}$$

where B represents the electrical resistivity of the charge whose electrical property is opposite to that of the surface of the soil particle in the electrical double layer, Q is the cation exchange capacity per unit soil pore, and BQ is the electrical resistivity of the electrical double layer on the surface of the soil particle.

The electrical resistivity model proposed by Waxman and Smits (1968) considers the effect of the electrical conductivity of soil particles on the electrical resistivity of the soil; thus, the electrical resistivity model proposed by Waxman and Smits (1968) is applicable to non-saturated cohesive soils.

In addition to soil particles and pore water, there is a third conductive propagation path for cohesive soils, i.e., the series-coupled soil–water propagation path (Rhoades and Schilfgaarde, 1976). Based on the aforementioned 3 conductive propagation paths for cohesive soils, the following equation for the model for the electrical resistivity of non-saturated cohesive soils has been deduced (Zha et al., 2006):

$$\rho = \left[ns_r - F^{'}\frac{\theta^{'}}{1+\theta^{'}}BQ + \frac{ns_r - F^{'}\frac{\theta^{'}}{1+\theta^{'}}}{\rho_w} + \frac{F^{'}\left(1+\theta^{'}\right)BQ}{1+BQ\rho_w\theta^{'}} \right]^{-1} \tag{4}$$

where $F^{'}$ is the conductive structure coefficient (the ratio of the width of the series-coupled soil–water path to the side length of the entire soil body) and $\theta^{'}$ is the volumetric water content of the parallel-coupled soil–water part.

The electrical resistivity model proposed by Zha et al. (2006) considers the effect of conductive paths and organically combines the electrical resistivity of a soil with factors such as the degree of saturation, porosity, electrical resistivity of the pore water, soil particle composition, soil structure and electrical double layers on the surfaces of soil particles, thereby rendering this model for the electrical resistivity of non-saturated cohesive soils more reasonable.

The equations that describe the relation between the electrical resistivity of a soil sample and the unfrozen water content and that between the electrical resistivity of a soil sample and the ice content are as follows (Fortier et al., 1994, 2008):

$$\frac{\rho}{\rho_{uw_o}} = e^{-w_{uw}/w_{uw_o}} \tag{5}$$

$$\frac{\rho}{\rho_{i_o}} = \left(\frac{w_i}{w_{i_o}}\right)^a \tag{6}$$

where ρ is the electrical resistivity ($\Omega\cdot$m), ρ is the unfrozen water content (%), w_i is the ice content (%), $\rho_{uw_0} = 12{,}820\ \Omega\cdot$m is the reference electrical resistivity for a reference unfrozen water content w_{uw_0} of 5%, $\rho_{i_o} = 1316\ \Omega\cdot$m is the reference electrical resistivity for a reference ice content w_{i_0} of 10%, and $a = 1.73$ is the exponent of the power law between the electrical resistivity and the ice content.

The electrical resistivity model proposed by Fortier et al. (2008) first considers the effect of the ice content of a soil on the electrical resistivity of the soil. Thus, the electrical resistivity model proposed by Fortier et al. (2008) is applicable not only to unfrozen soils but also to frozen soils. However, a frozen soil is a complex multiphase body, and there are many factors that affect a frozen soil. The electrical resistivity model proposed by Fortier et al. (2008) only considers the effect of the ice content of a soil on the electrical resistivity of the soil; in addition, the preset reference electrical resistivity value has no generality.

Angelopoulos et al. (2010) investigated frozen soil from Parsons Lake in the Northwest Territories of Canada using the electrical resistivity method and obtained the relationship between the electrical resistivity of the frozen soil and the ice content of the frozen soil. In this study, the electrical resistivity method was very usefully applied for frozen soil exploration. However, the data obtained using the electrical resistivity method were quite discrete and poorly correlated. In addition, the electrical resistivity method only considers the effect of the ice content on the electrical resistivity of the frozen soil and thus is limited.

2.2. Establishing a model for the electrical resistivity of frozen soils

During the freezing process, a fraction of the pore water of a soil undergoes a phase change; thus, the electrical resistivity characteristics of a frozen soil are different from those of an unfrozen soil. Based on the three-element electrical conduction model (Rhoades and Schilfgaarde, 1976) and the model for the electrical resistivity of unsaturated cohesive soils (Zha et al. 2006), the present study also assumes that there are 3 conductive paths (soil particles, ice–water mixtures and soil–ice–water mixtures, i.e., the gas propagation path is ignored) for a frozen soil and deduces the equation for the model for the electrical resistivity of frozen soils. Fig. 1 shows the conduction model for ice–water mixtures; it is