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Research Paper

Application of the Lambert W function for the impact of the exhaust gas turbocharger on nanoscale PM emissions from a TGDI engine

Matteo Cucchi, Stephen Samuel *

Department of Mechanical Engineering and Mathematical Sciences, Oxford Brookes University, Oxford, UK

HIGHLIGHTS

- Nano-scale PM from a TGDI engine is measured before and after the turbine.
- Variations in PM across the turbine are modelled by means of the Lambert W Function.
- The model predicts turbine-out PM with a coefficient of determination of 0.917.
- The input variables of the model are known to the ECU or can be measured on-board.
- The model could be implemented within engine performance simulation software.

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ABSTRACT

A phenomenological model based on the Lambert W Function is presented which is capable of describing the variations in nano-scale particulate matter (PM) number concentrations across the turbine stage of a turbocharged gasoline direct injected (TGDI) engine over a wide range of engine operating conditions with a coefficient of determination (CD) of 0.917. The model predictors are variables which are available to the engine control unit (ECU) either by means of stored maps or on-board measurements. The suitability of the proposed model for real-time usage within the ECU for vehicle calibration is discussed.

1. Introduction

Engine design over the recent years has been addressing the need to reduce the levels of pollutants emissions and fuel consumption while maintaining high levels of performance and drivability [1,2]. These goals have been achieved with the development of new, highly downsized, engines [3]. While the brake-specific emissions of the species regulated up to Euro-V European Emission Standards are reduced, i.e. the amounts of pollutants emitted per unit of power produced by the engine are lower for turbocharged gasoline direct injected (TGDI) engines when compared to previous generation, naturally aspirated (NA), port fuel injected (PFI) engines as a result of reduced displacement and fuel consumption [4], fuel direct injection causes a particulate matter (PM) emissions increase because of a much shorter time for mixture preparation [5], and because of wall wetting which results in liquid fuel burning from the cylinder and piston surfaces [6,7].

Previous research by the authors using a differential mobility spectrometer was the first to show that the exhaust gas turbocharger installed on a TGDI engine has itself a strong influence on the PM number count from the engine [8]. These experimental results stressed the fact that the design of new engines with the goal of PM emissions reduction should not focus only on the well-known, in-cylinder, combustion-related phenomena but needs to address also the engine peripherics which are present on the exhaust line of the engine. Research showed that PM number counts measured before and after the turbocharger over a wide range of engine operating conditions increased up to +297% across the turbine stage in the direction of flow. Considering the impact of PM on human health [9] and considering the new, stringent limits on PM number emissions introduced by the Euro-VI European Emission Standards for GDI engines in addition to the already existing restrictions on PM mass emissions [10–12], it is important that engine manufacturers are able to accurately account for the contribution of the exhaust gas turbocharger on the PM number emissions from their TGDI engines. This paper details a phenomenological model based on the Lambert W function that is capable of describing the variations in nanoscale PM number concentrations across the turbine

* Corresponding author. Tel.: + 44 1865 483513.
E-mail address: s.samuel@brookes.ac.uk (S. Samuel).

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stage. The proposed model is simple in its formulation yet it allows to predict the change in PM number emissions across the turbine over a wide range of engine operating conditions with a coefficient of determination (CD) of 0.917. It therefore could be embedded within the ECU for control or diagnostics purposes of a real engine, or it could be coupled in a simulation environment with an engine model in order to simulate the impact of the exhaust gas turbocharger on the nanoscale PM emissions of the engine if engine-out emissions are predicted by the engine model or imposed from experimental data.

2. The Lambert W function

The Lambert W function, first named after Johann Heinrich Lambert [13] by [14] and also known as the Omega function, is defined to be the multi-valued inverse of the function $w \mapsto we^w$. In other terms, $W(x)$ is defined to be the function satisfying:

$$W(x)e^{W(x)} = x \quad (1)$$

The graph of $W(x)$ for x real is shown in Fig. 1. For $-1/e \le x < 0$ there are two possible real values of $W(x)$, one single real value for $x \ge 0$, and no real value for $x < -1/e$. Following the work by [14], $W_0(x)$, also known as the principal branch of the W function, is the branch satisfying $W(x) \ge -1$. The branch satisfying $W(x) < -1$ is denoted as $W_{-1}(x)$.

There are several applications of the Lambert W Function in Mathematics, Physics, Chemistry, and other sciences. Such a function has been used in the solution of linear constant-coefficient delay equations and in the general analysis of time-delay systems, which is likely its most remarkable application [15–20]. It was also successfully used in enumeration and combinatorics [21], to model combustion [22], in enzyme kinetics [23], in the calculation of exchange forces between the two nuclei in the hydrogen molecular ion H_2^+ [24,25], in the solution of the Richards' equation for water movement in soil [26] and in the improvement of the Aronofsky model for oil recovery from fractured water-wet reservoirs [27], in the solution of the Volterra equations for population growth [28,29], in the study of the spread of disease [30] and in the analysis of the dynamic physiology governing cardiorespiratory stability [31]. Other notable applications can be found in the derivation of the outflow speed and the mass loss rate of the solar wind of plasma particles ejected by the sun [32], in the study of water-wave height in oceanography, in the calculation of the potential of semi-infinite plane capacitor fields, and in the evaluation of Wien's displacement constant in Wien's displacement law [33], and ultimately in several electronics problems [34–38].

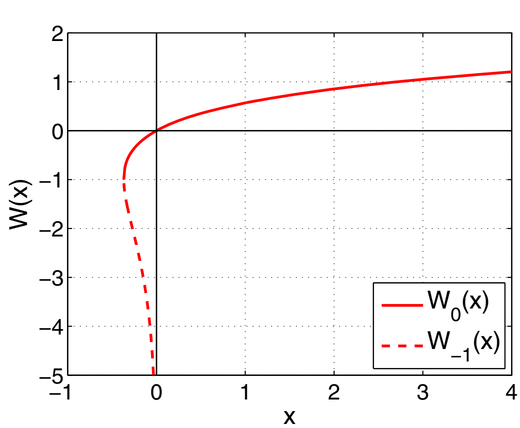

Fig. 1. The two real branches of the Lambert W function: $W_0(x)$ and $W_{-1}(x)$.

Apart from its simplicity, one of the merits of the Lambert W Function lies in the fact that well-tested algorithms and numerical methods with arbitrary precision are available for the evaluation of its values and branches [39], based on both analytical approximations [40], and on Newton's and Halley's iteration schemes [41]. This makes it suitable for formulae and expressions which include the Lambert W Function to be implemented within a calculation device like an ECU, as they can be evaluated on-board and their output used for control and diagnostics purposes. The following paragraphs give a summary of the experimental outcomes from the authors' test campaign and outline the phenomenological model developed by the authors, which is based on the Lambert W Function that is capable of predicting those results.

3. Experimental results

A differential mobility spectrometer was used to measure the nanoscale PM number emissions count from a TGDI engine before and after the turbine stage in order to analyse the influence of the turbine itself on the nanoscale PM number emissions count. Controlled, repeated experiments were performed for various engine steady-state operating conditions (defined by engine speed and engine load) over a full-factorial test matrix of 3 engine speeds (1500, 2500, and 3500 rpm) and 4 engine loads (20, 50, 80, and 110 Nm). The engine manufacturer calibration and commercially-available fuel from the same batch were used in order to represent real-life operating conditions. For repeatability analysis, three independent measurements are taken at both sampling locations (before and after the turbocharger) on separate days for each test point. All experiments are performed after a complete engine warm-up which lasts for 20 minutes to allow for engine stabilisation [8]. PM number count measurements are taken sequentially before and after the turbine for every test point with a differential mobility spectrometer. The measurement output is a graph of particle number count versus particle diameter D_p, as a function of time. The particle size distribution is then averaged over the duration of the measurement to yield a plot of mean spectral concentration versus particle diameter, and integrated over particle size in order to calculate the total number count emissions. Fig. 2 shows an exemplary comparison of PM particle size distributions before and after the turbine for one of the operating points under investigation (2500 rpm – 80 Nm). As it can be seen, there are substantial differences in particle concentrations for the two measurement locations, especially for particle diameters around 10 and 100 nm, resulting in a larger particle number count at the turbine outlet with respect to its inlet.

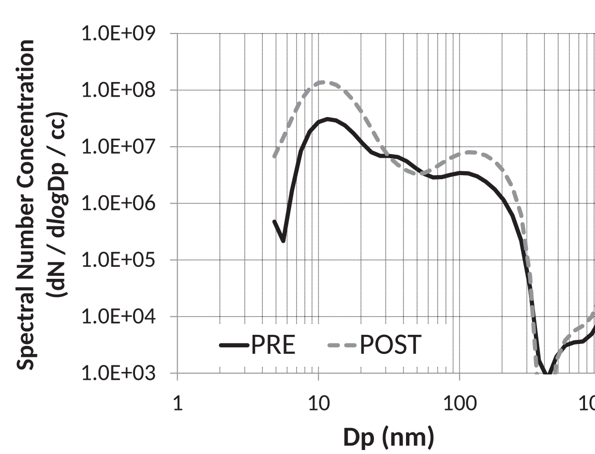

Fig. 2. Exemplary comparison of PM particle size distributions before and after the turbine for one of the operating points under investigation.