



Research Paper

Simplified analysis of heat and mass transfer model in droplet evaporation process

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HIGHLIGHTS

- Differential equations were developed in order to explain droplet evaporation.
- The model shows great coherency with experimental results of our predecessors.
- Relative humidity has a great effect on the decrease of the droplet radius.
- Large Ste condition leads to error of about 15%–20%.

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ABSTRACT

A simplified model on single droplet evaporation into a stagnant gas space is established. The mathematical model is based on molecule diffusion on the droplet surface subjected to boundary conditions of constant temperature and conduction heat transferring on the external surface of the spherical droplet. Constant thermal properties are assumed throughout the analysis for the droplet. Non-linear empirical equations are applied to the ODEs in the numerical solution to expand its range of applicability. The effects of variables, such as surrounding temperature and humidity, and droplet initial radius were investigated. Furthermore, a comparison is made between the present study and the numerical or analytical solution given in the literature. The results show that the present model can give better results than other numerical or analytical solutions presented in the literature.

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1. Introduction

Droplet evaporation is of importance in, for example, process operations, liquid hydrocarbon combustion, and refrigeration. Evaporation of a single isolated liquid droplet into a stagnant gas space has long been a subject of study; recent reviews on this activity have been given by Erbil [1] and Tso and Chao [2]. Numerical simulation of droplet dynamics and heat and mass transfer processes is a particularly challenging problem. Momentum, mass and energy interchanging between the gas and liquid phases are crucial for the accurate prediction of the interacting droplet evaporation phenomena that are addressed in a number of research papers. (Davies [3]; Sergei [4]; Maatar et al. [5]; Gumulya et al. [6]; Oberman and Farrell [7]). Single-droplet computational fluid dynamics (CFD)

simulations (Chiang et al. [8]; Raghuram et al. [9]; Stamoudis et al. [10]) and experimental investigations (Vysokomornaya et al. [11]; Gao et al. [12]; Volkov et al. [13]) have yielded correlations that can be used in numerical calculation of evaporation properties of droplet. Strotos et al. [14] examine the evaporation of single- and multi-component droplets under forced convection conditions in a hot air environment using the VOF methodology. The energy and concentration equations inside the liquid and the gaseous phases for both liquid species and their vapor components are additionally solved, coupled together with a model predicting the local vaporization rate between the liquid and the surrounding gas, but the model shows good agreement under convective flow conditions. Large eddy simulations of very high speed evaporating sprays issue are performed with a hybrid Eulerian–Lagrangian mathematical/computational model and was conducted by Irannejad and Jaber [15], the focus is on the high speed evaporating sprays and they found that the gas velocity and turbulence generated by the spray are very significant to droplet evaporation. Choo, et al. [16] investigated the evaporation of water and ethanol droplets with and without laser excitation. They analyzed the influence of the external disturbance factor on the droplet evaporation. They found that the

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evaporation rate and temperature rise are much higher for the water droplets in comparison with the ethanol droplets at the relative similar conditions. The variation of evaporation rate is not monotonous in different parameter ranges. Kuznetsov et al. [17] investigated the evaporation on fixed water droplet containing a single nontransparent solid inclusion placed in gaseous environment at high-temperature. Cheng et al. [18] presented a comprehensive mathematical model of vacuum flash evaporation cooling based on the film penetration theory. The droplet temperature and diameter after droplet flash evaporation were obtained numerically, and then analyzed. The effect of droplet flash evaporation on the surface temperature distribution, heat flux density, and heat transfer ratio of different mechanisms was also studied based on the model. Esteve [19] analyzed the physics of droplet evaporation on different surfaces, and some existent models are implemented and compared with current experimental results. It has been realized that the driven variable of evaporation time is the mass transfer rate. The evaporation process does not depend on the geometry of the wetted surface; the model designed for smooth surfaces work also in rough surfaces as long as the droplet is considered as a cylinder instead of a spherical cap. Tonini and Cossali [20] developed a novel evaporation model for multi-component spherical drop based on the Stefan–Maxwell equations; the evaporation rate predicted by the new model is compared with the predictions obtained by previous models based on Fick's law approximation, under steady-state isothermal conditions for a wide range of gas and drop temperatures and compositions. The predictions from the isothermal models generally underestimate the total evaporation rate, particularly at higher temperatures, for different drop compositions.

Most of the relevant models are focus on solving the problem of Navier Stokes equations. Indeed, they get conclusions very close to the experimental data. But the mathematical treatment is very complicated, so it is not conducive to engineering application. Here, based on the basic thermodynamics theory, a relatively simple but high accuracy model is derived.

At the very beginning, researchers usually start with the heat and mass transfer equations under different boundary conditions with an effort to seek for the analytical solutions. We can easily find the general solution for the transient heat and mass transfer equation in sphere coordinate:

$$T(r, t) = A + B \left[\frac{\sqrt{\alpha t}}{r} \exp\left(-\frac{r^2}{4\alpha t}\right) - \frac{\sqrt{\pi}}{2} \operatorname{erfc}\left(\frac{r}{2\sqrt{\alpha t}}\right) \right] \quad (1)$$

where A , B are integration constant. This solution shows singularity when $r \rightarrow 0$ and cannot satisfy many boundary conditions. In order to solve the complex equations, some scholars [21] expanded the solution as a series about a small parameter related to the boundary conditions (perturbation method). This method can solve some practical problems in a proper way. Many other methods are concentrated on the mathematical treatment of the parabolic equations (see Fatoorehchi et al. [22–24])

The present study considers the vaporization of a liquid droplet at rest relative to gaseous surroundings of finite extent. We only consider the evaporation rate of a single droplet. A spherically symmetric quasi-steady diffusion-controlled model has been employed. The model contains the following essential features: simultaneous heat and mass transfer in the liquid droplet, phase change across the gas/liquid interface, and diffusion rate depends on the thermal properties of the surrounding gas. We have formulated the governing equation from the perspective of molecular diffusion without considering the convective and radiative heat transfer effect. The effects of radiation on vaporization of droplet can be found in Brewster [25]. Solving equations numerically for different condition lead to our expected solution with previous experimental data.

2. Mathematical formulation

2.1. Size change of a single droplet

Droplet evaporation is inherently a time-dependent phenomena with the droplet radius decreasing with time due to mass loss. As fine droplets stay in a stable vapor phase, the evaporation process occurs. Due to the surface tension effect, the spherical droplets and contact surface are formed. If the interfacial mass transfer resistance is neglected, it can be assumed that the droplet interface is in equilibrium, which corresponds to the equilibrium concentration of water. Two phases are separated by the phase separation boundary $r = R(t)$, which will move inward as evaporation progresses. Due to the concentration difference, the water molecules move from the interface toward the vapor space. Because of the large external space, the influence of water droplets on the content of water vapor can be neglected, and the ambient is considered to be in a quasi-steady state. When water content in the ambient air is in a steady state, the particle fluxes at the surface of the selected droplet – that is, the rate of concentration per unit area per unit time – are given by Fick's first law of diffusion:

$$J = -D_v \frac{\partial c}{\partial r} \quad (2)$$

where $\partial c / \partial r$ is the concentration gradient of the selected droplet. For droplet, replacing the diffusing particles by their concentration variation simplifies the derivation of the diffusion equations. The rate of mole reduction of the droplet with time is given by

$$\dot{n} = 4\pi r^2 D_v \frac{\partial c}{\partial r} \quad (3)$$

The radial gradient of vapor concentration, $\partial c / \partial r$, is determined by the following equation:

$$\frac{\partial c}{\partial r} = \frac{\dot{n}}{4\pi r^2 D_v} \quad (4)$$

Integrating Eq. (4) gives:

$$c_r - c_\infty = \frac{\dot{n}}{4\pi D_v} \int_r^\infty \frac{dr}{r^2} = -\frac{\dot{n}}{4\pi r D_v} \quad (5)$$

Under low partial pressure, the water vapor can be regarded as an ideal gas. Eq. (5) can be expressed as follows by the relation $c = p / RT$, where c_r and c_∞ are vapor concentrations at radius r and ∞ , respectively. Substituting $c = p / RT$, Eq. (5) becomes

$$\dot{n} = \frac{4\pi r D_v}{R} \left(\frac{p_\infty}{T_\infty} - \frac{p_r}{T_r} \right) \quad (6)$$

where p_∞ and T_∞ are surrounding pressure and temperature respectively. On the other hand, concentration change rate of single droplet can also be expressed as

$$\dot{n} = \frac{1}{M} \frac{dm}{dt} = \frac{\pi \rho_d D_d^2}{2M} \frac{dD_d}{dt} \quad (7)$$

where ρ_d and D_d represent the density and diameter of droplet, respectively, and M being molar mass of water. From Eqs. (6) and (7), drop size change can be expressed as

$$\frac{dD_d}{dt} = \frac{8MrD_v}{\rho_d D_d^2 R} \left(\frac{p_\infty}{T_\infty} - \frac{p_r}{T_r} \right) \quad (8)$$

When a particle approaches the ending of evaporation, its size will likely be less than the mean free path. For this condition, the rate of particle diminish is governed by the rate of random

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