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Unsaturated shear strength of a silty sand

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ABSTRACT

Series of single stage, consolidated drained direct shear tests under different net normal stresses and matric suctions were carried out to evaluate the unsaturated shear strength behavior of a silty sand. A back pressure shear box was used to measure and control pore-air and pore-water pressures. The experimental data show that matric suction increases shear strength of the soil significantly following a highly non-linear relationship. An increase in net normal stress results in a higher shear strength and the shearing behavior changes gradually from a peak shear strength followed by a strain-softening behavior to a strain-hardening behavior. A comparison between experimental shear strength measurements and predicted unsaturated shear strength values was made and the results are presented in the paper.

1. Introduction

The shear strength of a soil is required in the stability assessment of slopes and embankments. The Mohr–Coulomb theory, utilizing the effective stress state, is commonly used for describing the shear strength of saturated soils. However, since soils are seldom fully saturated and in order to capture the real behavior of an unsaturated soil, analyses have to be carried out with the consideration of a possible increase in shear strength due to matric suction.

The unsaturated shear strength can be determined experimentally in the laboratory using a modified direct shear apparatus or a modified triaxial apparatus. Escario and Saez (1986) showed that the effective friction angle of some soils increases slightly with increasing matric suction. However, for most practical purposes, the effective friction angle may be considered constant for suction values less than 500 kPa, which is often the range of practical interest for geotechnical and geo-environmental engineering (Vanapalli et al., 1996). Experimental studies showed that the variation of shear strength with respect to matric suction is non-linear (e.g., Escario and Saez, 1986; Gan et al., 1988; Escario and Juca, 1989; Melinda et al., 2004). The non-linearity of the unsaturated shear strength envelope was related to the Soil–Water Characteristic Curve (SWCC). As the soil remains saturated up to the air-entry value (AEV), a linear relationship between shear strength and matric suction is in general obtained. As the soil starts to desaturate beyond the AEV, the relationship between shear strength and matric suction becomes non-linear (Vanapalli et al., 1996). Donald (1956) performed direct shear tests on sands and showed that the shear strength with respect to matric suction can reach a plateau or even decreases when the degree of saturation decreases considerably.

The objective of the study presented in this paper is to investigate the effect of matric suction on the shear strength characteristic and behavior of a recompacted silty sand using a back pressure shear box. A comparison of the experimental shear strength data and the prediction of the unsaturated shear strength of the soil is also made.

2. Shear strength of unsaturated soils

Bishop and Blight (1963) proposed a shear strength equation for unsaturated soils by incorporating the modified effective stress expression into the classical Mohr–Coulomb failure criterion.

$$\tau = c' + [(\sigma_n - u_a) + \chi(u_a - u_w)]\tan(\phi') \quad (1)$$

where τ = unsaturated shear strength; c' = effective cohesion; ϕ' = effective friction angle; $(\sigma_n - u_a)$ = net normal stress; $(u_a - u_w)$ = matric suction; and χ = effective stress parameter.

The effective stress parameter (χ) varies from 1 for a saturated condition to 0 for a completely dry condition but depends on so many factors, making it difficult to be estimated. The equation is also based on the assumption that the effective cohesion and the effective friction angle are independent of matric suction.

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The shear strength of unsaturated soils using the independent stress state variable approach by Fredlund et al. (1978) can be written as:

$$\tau = c' + (\sigma_n - u_a)\tan(\phi') + (u_a - u_w)\tan\left(\phi^b\right) \tag{2}$$

where ϕ^b = angle indicating a change in shear strength related to matric suction.

The first two terms describe the classical Mohr–Coulomb criterion when the pore-air pressure is equal to the pore-water pressure. The third term introduces ϕ^b as an angle indicating a change in shear strength due to matric suction. Up to the AEV where the soil remains essentially saturated, ϕ^b is reported to be equal to ϕ'. Beyond the AEV when the soil starts to desaturate, ϕ^b is reported to decrease with an increase in matric suction and the relationship becomes non-linear (e.g., Gan et al., 1988; Vanapalli et al., 1996; Rassam and Williams, 1999; Lee et al., 2005).

2.1. Models to predict the unsaturated shear strength

Several equations to describe the unsaturated shear strength exist. These equations usually consist of two parts. The first part shows the saturated shear strength and the second part shows the increase in shear strength due to an increase in matric suction. As the SWCC is the key function for unsaturated soils, many equations for the unsaturated shear strength are related to the SWCC.

Unsaturated shear strength equations can be classified into fitting and prediction type equations which have been proposed by numerous researchers (e.g., Vanapalli et al., 1996; Oberg and Sallfors, 1997; Khalili and Khabbaz, 1998; Tekinsoy et al., 2004). At least one fitting parameter is introduced for the fitting type equations to match the calculated data to the unsaturated shear strength measurements. Unsaturated direct shear strength measurements involve expensive equipment and long testing time which makes it expensive and time consuming. For the prediction type of equations, no unsaturated shear strength measurements are needed. The equations usually consist of the saturated shear strength parameters and the SWCC, or parameters of the SWCC. These equations are attractive for a first approximation of the unsaturated shear strength as even the SWCC can be estimated. SWCCs can be estimated using physicoempirical models based on the grain-size distribution or through correlations between classification of soil properties (e.g., grain-size distribution and Atterberg limits) which provide approximate parameters for SWCC equations (e.g., Zapata, 1999; Fredlund et al., 2002). These estimated SWCCs are satisfactory in many situations (Zapata, 1999; Fredlund, 2006).

In this study, experimentally obtained shear strength data of a silty sand are compared with results from a number of prediction type equations (Table 1). Even though some of the equations were developed and reported suitable for conditions which differ from the current soil condition, all these equations were used in the analyses to assess which equation performs best for the silty sand used in this study. In order to compare all the equations consistently over the suction range where shear strength measurements were available, assumptions had to be made for some of the equations which include:

- Increase in the shear strength with respect to matric suction up to the AEV equal to ϕ' if the equation is not valid below the AEV.

Table 1
Published equations for the unsaturated shear strength—prediction type of equations.

Author	Shear strength equation—prediction type	
Goh et al. (2010)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + (u_a - u_w)\tan\left(\phi^b\right)$ where $\phi' = \phi^b$ if $(u_a - u_w) < \text{AEV}$ $\tau = c' + [(\sigma_n - u_a) + \text{AEV}]\tan(\phi') + [(u_a - u_w) - \text{AEV}]b\Theta^{\kappa}\tan(\phi')$ *if* $(u_a - u_w) \geq \text{AEV}$	$\kappa = [\log(u_a - u_w) - \log(\text{AEV})]^y$ For drying: $y_d = 0.502\ \ln(I_p + 2.7) - 0.387$ $b_d = -0.245\{\ln[n_d(I_p + 4.4)]\}^2 + 2.114\{\ln[n_d(I_p + 4.4)]\} - 3.522$ For wetting: $y_w = 3.55y_d - 3.00$ $b_w = 0.542b_d\left(\frac{n_d}{n_w}\right) + 0.389$
Sheng et al. (2008)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + (u_a - u_w)\tan\left(\phi^b\right)$ where $\tan\left(\phi^b\right) = \tan(\phi')$ *if* $(u_a - u_w) < (u_a - u_w)_{sa}$ where $\tan\left(\phi^b\right) = \tan(\phi')\left[\frac{(u_a - u_w)_{sa}}{(u_a - u_w)} + \left(\frac{(u_a - u_w)_{sa} + 1}{(u_a - u_w)}\right)\ln\frac{(u_a - u_w) + 1}{(u_a - u_w)_{sa} + 1}\right]$ *if* $(u_a - u_w) > (u_a - u_w)_{sa}$	$(u_a - u_w)_{sa}$: Saturation suction
Garven and Vanapalli (2006)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + \Theta^{\kappa}(u_a - u_w)\tan(\phi')$	$\kappa = -0.0016I_p^2 + 0.0975I_p + 1$
Tekinsoy et al. (2004)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + \tan(\phi')(\text{AEV} + P_{at})\ln\left[\frac{(u_a - u_w) + P_{at}}{P_{at}}\right]$	
Aubeny and Lytton (2003)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + f_1(\theta)(u_a - u_w)\tan(\phi')$ *if* $S = 100\% : f_1 = \frac{1}{\theta}$ *if* $85\% \leq S \leq 100\% : f_1 = 1 + \frac{S - 85}{15}\left(\frac{1}{\theta} - 1\right)$ *if* $S \leq 85\% : f_1 = 1$	
Vanapalli et al. (2000)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + \Theta^{\kappa}(u_a - u_w)\tan(\phi')$	$\kappa = f(I_p)$
Khalili and Khabbaz (1998)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + (\chi)(u_a - u_w)\tan(\phi')$	$\chi = \left[\frac{(u_a - u_w)}{\text{AEV}}\right]^{-0.55}$
Bao et al. (1998)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + (\zeta)(u_a - u_w)\tan(\phi')$	$\zeta = \left[\frac{\log(\text{RS}) - \log(u_a - u_w)}{\log(\text{RS}) - \log(\text{AEV})}\right]$
Oberg and Sallfors (1997)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + (S)(u_a - u_w)\tan(\phi')$	
Vanapalli et al. (1996)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + \left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)(u_a - u_w)\tan(\phi')$	
Lamborn (1986)	$\tau = c' + (\sigma_n - u_a)\tan(\phi') + (\theta)(u_a - u_w)\tan(\phi')$	