Minerals Engineering 109 (2017) 21–31

Contents lists available at ScienceDirect

Minerals Engineering

journal homepage: www.elsevier.com/locate/mineng

The efficiency of copper ore comminution: A thermodynamic exergy analysis

Wilhelm Jacob [a], Daniel R. Cooper [c], Timothy Gutowski [b], Jorge Ramos-Grez [a],*

[a] *Mechanical and Metallurgical Department, Pontificia Universiad Católica de Chile, Avenida Vicuña Mackenna 4860, Macul 7820436, Santiago, Chile*
[b] *Massachusetts Institute of Technology, 77 Massachusetts Avenue, Room 35-234, Cambridge, MA 02139, USA*
[c] *University of Michigan, 2458 George G. Brown Laboratory, 2350 Hayward, Ann Arbor, MI 48109-2125, USA*

ARTICLE INFO

Article history:
Received 30 June 2016
Revised 22 February 2017
Accepted 24 February 2017
Available online 8 March 2017

Keywords:
Comminution
Exergy
Dry ball mill
Minimum work energy

ABSTRACT

This study analyzes the exergy efficiency of comminution using a laboratory sized dry ball mill and a copper mineral ore from central Chile. In this research, we develop a minimum work index from uniaxial compression tests results on the ore fitted to Morrell's comminution energy equation using Hukki's constant mineral parameter. An exergy analysis is then performed on a laboratory sized dry ball mill by considering the surface energy variation for different ore sizes, the obtained Hukki-Morrell fitted relationship from the compression tests of the ore and the minimum inertial energy required during a dry-ball milling process. The theoretical minimum work needed to crush the rock is first calculated by comparing the difference in surface energy between the incoming ore and the outgoing crushed rock. However, this analysis (based on the creation of new surfaces) suggests an impractical low minimum work of little use as a benchmark. Instead, use of the Hukki-Morrell relationship evaluated at the resulting ore particle sizes suggests a practical benchmark. Using this benchmark for the laboratory's dry ball mill process renders an efficiency of approximately 3%. One could use the heat from the outgoing crushed ore to, theoretically perform work, but the heat flows through at a low temperature that this change would only increase the efficiency to 3.1%. We also consider the inertial energy needed to drive the ball mill employing it as a new benchmark for our efficiency calculation. The latter raises the efficiency of the dry ball milling to an average 23%. Finally, electrical energy measurements and inertial calculations on operating an empty versus loaded ball mill suggest that the efficiency could be increased by ensuring correct machinery scaling (diameter of the drum shell), ball load and a properly maintained transmission system.

1. Introduction

Comminution is the breaking of larger rocks into smaller ones through crushing, grinding, or other processes. In copper mining, it is used to crush the ore to a fine powder. In this powder form, froth flotation tanks can be used to separate the copper concentrate from the gangue (the remainder of the mined material). Since the first reported use of ball mills in 1870 (Lynch and Rowland, 2005), several improvements have been made to the process by studying the different parameters such as the ball and ore load ratio and quantity, ball diameters, performance speeds and the viscosity of the slurry.

The aim of this research is to study the efficiency of the comminution process through the first and second laws of thermodynamics. The first law concerns the conservation of energy, which for a steady state manufacturing process translates to a balance among all energy flows into and out of the system. The second law considers the entropy of the system, which is a statistical measure of the dispersion of energy. This leads to the concept of available work, or exergy, which is equal to the maximum amount of work that can be obtained as a result of a given system coming into equilibrium with a reference state defined by its (constant) temperature, pressure and chemical potentials. Exergy is not a conserved property and the more exergy is destroyed the more irreversible and inefficient the process under consideration is. An exergy analysis allows the definition of a benchmark thermodynamic minimum work requirement, which is useful when evaluating efficiencies. An exergy analysis also allows both material and energy flows to be directly compared (Rosen et al., 2008). Therefore, adopting the latter evaluation analysis of the system provides an optimization limit at which the process could not be, theoretically, further improved. A more detailed discussion on exergy,

---

* Corresponding author at: Mechanical and Metallurgical Department, Pontificia Universiad Católica de Chile, Avenida Vicuña Mackenna 4860, Macul 7820436, Santiago, Chile.

E-mail address: jramos@ing.puc.cl (J. Ramos-Grez).

http://dx.doi.org/10.1016/j.mineng.2017.02.011
0892-6875/© 2017 Elsevier Ltd. All rights reserved.

entropy, and the second law of thermodynamics can be found elsewhere (Bakshi et al., 2011).

Much research has been done in the area of energy analysis of systems that undergo material transformations, especially from a manufacturing processes perspective and mostly focused on first law analysis (Bakshi et al., 2011). An interesting exergy analysis on the subject of comminution was done by Petela (1984) and further on by Alvarado et al. (1998), however both their models are focused onto the volume energy of comminution and not much attention is paid to the associated surface energy changes. Whereas other researchers such as Ballantyne and Powell (2014), Musa and Morrison (2009), Stamboliadis (2007) and Tromans (2008) has centered on this latter issue.

The exergy analysis established in the present study considers the chemical exergy of the copper ore, changes to its surface area, the physical exergy of material flows due to temperature changes, the electrical work consumed, and the destroyed exergy due to irreversibilities in the overall process. Alongside a theoretical minimum work for comminution, we also established a "practical minimum work" based on a laboratory scale uniaxial compression tests. The results obtained from the uniaxial compression tests are then fitted to a comminution energy relationship developed elsewhere (Rittinger, 1867; Kick, 1885; Hukki, 1962; and Morrell, 2004), and compared against a comminution minimum work estimation previously developed by Bond (1952). Following this, the latter is compared against an estimate of the inertial work of a dry ball mill, considered as the minimum work for the machinery *per se*. Further on, this methodology is applied to the exergy efficiency assessment of the laboratory sized dry ball mill.

2. Theoretical background

Previous attempts to model comminution with regards to the theory of exergy have informed the methodology of this research. Subsequently, a brief review of both is presented here.

2.1. Comminution theory

In order to estimate the amount of energy required to crush ore from one initial diameter to a final diameter, differential forms of the Rittinger, Kick and Bond models are integrated from x_{feed} to $x_{product}$. Eq. (1) presents the common differential form of these models where dE denotes the differential amount of energy needed to reduce the ore size, x refers to the evaluated ore size, K is the mineral parameter which distinguishes ores from one source origin to another and is not necessarily the same value for the different energy relationships, and m is a dimensionless exponent. This latter exponent equals 2 for Rittinger's comminution energy model (Rittinger, 1867), 1 for Kick's (Kick, 1885), and 1.5 for Bond's (Bond, 1952).

$$dE = -K \cdot x^{-m} dx \tag{1}$$

The integral of Eq. (1) takes the following form when evaluated between two particles sizes, from x_f to x_p,

$$\Delta E = -\frac{K}{1-m} \cdot \left(x_p^{1-m} - x_f^{1-m}\right) \tag{2}$$

In the particular case of Bond comminution model, m is equal to 1.5, and the latter equation takes the known form for the comminution energy between average feed particle size of the mineral ore to product particle size,

$$\Delta E = 2K \cdot \left(\frac{1}{\sqrt{x_p}} - \frac{1}{\sqrt{x_f}}\right) \tag{3}$$

On the other hand, Hukki noticed that the exponent m in Eq. (1) varied throughout the ore size range (Hukki, 1962). Lynch (1977) and later on Kapur and Fuerstenau (1987), further explained that the curve described by Hukki could be represented in its differential form by Eq. (4). They explained that what Hukki described differs from the previous comminution models in that the exponent m, corresponds to a function $f(x)$ taking into account the size, x, of the ore particles.

$$dE = -K \cdot x^{-f(x)} dx \tag{4}$$

Following Hukki's observation, Morrell proposed that the mineral parameter K also varies with ore size through a characteristic function $C(x)$ but maintained a material constant K' (Morrell, 2004). The same study went further proposing that the integral of this differential form (Eq. (4)), could be represented as shown in Eq. (5), when evaluated again between the feed size (x_f) and the product size (x_p) of the ore particles,

$$\Delta E = K' \cdot C(x) \cdot \left(x_p^{-h(x_p)} - x_f^{-h(x_f)}\right) \tag{5}$$

and where the exponent $h(x)$ corresponds to a function of the ore size, x. Morrell (2006) went onto suggesting that the latter function could take the form $h(x) = a + xb$, with parameters a and b fitted experimentally.

In this study, we made use of Morrell's equation for estimating the energy consumption within a given size reduction range (i.e. feed to product size), as illustrated in Eq. (6), however keeping the mineral parameter K fixed as in Hukki's differential representation.

$$\Delta E = K \cdot \left(x_p^{-(a+bx_p)} - x_f^{-(a+bx_f)}\right) \tag{6}$$

The selection of this semi-empirical correlation, referred by us as the Hukki-Morrell relationship and the estimation of a minimum comminution energy value will be important throughout the course of this study. We will make use of the Bond comminution energy (Eq. (3)), as it is elsewhere a used ubiquitous model, to provide a proper benchmark to compare estimates of the comminution energy from Eq. (6).

2.2. Conducting an exergy assessment

In order to conduct an exergy assessment, a thermodynamic reference state is required, as exergy of a system depends on the differences between the thermodynamic state of the system and its environment (Bakshi et al., 2011). This reference state can be defined as being at ambient temperature, T_o, ambient pressure, P_o, and of an idealized chemical composition such as that defined by Szargut et al. (1987).

The exergy of a system contains both a physical and a chemical exergy as seen in Eq. (7). The physical exergy depends on temperature and pressure changes - as shown in Eq. (8) where, Cp is the heat capacity of the material at constant pressure. In this analysis pressure changes are not considered.

$$Exergy_{sys} = Exergy_{ch} + Exergy_{ph} \tag{7}$$

$$Exergy_{ph} = Cp(T - T_0) - T_0 Cp \ln\left(\frac{T}{T_0}\right) \tag{8}$$

Additionally, surface energy of ore the particles is considered as part of the chemical exergy. Under reversible thermodynamic conditions (i.e. adiabatic and quasi-static process) the energy for comminution would corresponds purely to surface energy difference due to the increase in surface area during separation of the ore into smaller particles. Having these considerations, the general exergy rate balance analysis of an open system is then given by Eq. (9).