Building and Environment 126 (2017) 98–106

Contents lists available at ScienceDirect

Building and Environment

journal homepage: www.elsevier.com/locate/buildenv

Analytical model for the moisture absorption in capillary active building materials

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ARTICLE INFO

Article history:
Received 19 June 2017
Received in revised form
6 August 2017
Accepted 12 September 2017
Available online 21 September 2017

Keywords:
Capillary active materials
Water uptake test
Free boundary problem

ABSTRACT

Isothermal absorption of moisture in a semi-infinite porous body is described with a diffusion law as a free boundary problem in which the porous domain is divided into a nearly saturated and a nearly dry region. This problem is solved via an analytical approach by assuming the diffusivity as a step function of the volumetric water content. The analytical solution is then applied to modeling the behavior of two real building materials during a water uptake test. It is shown that the model is able to explain the absorption process adequately, and that a good agreement with the measured data is obtained. Finally, a novel method for the inverse determination of the diffusivity via water uptake tests is proposed.

1. Introduction

An exact understanding of the moisture transfer in porous building materials is required in order to design reliable building components and prevent the risk of moisture damages. The increasing interest in this topic is proved by numerous recent studies concerning both modeling approaches and experimental methods, see e.g. Refs. [1–15]. For what concerns the characterization of material properties, a set of well established experiments are already available. One of these is the so called water uptake test, which is used to determine the water uptake coefficient [3–8] and the water diffusivity [9–11]. In this experiment, moisture arises due to capillary suction inside an initially dry material sample. The purpose of this study is on the one hand to present and validate an analytical model able to explain this absorption behavior, on the other hand to introduce a novel experimental method for determination of the diffusivity.

It has been observed that the capillary moisture transfer obeys a diffusion law, in which the dependent variable is the volumetric water content, see e.g. Refs. [3,10]. Hence, the time dependent water content distribution can be determined if the material diffusivity and the boundary and initial conditions are known. Even though this problem can be easily solved via numerical methods (see e.g. Refs. [1,2]), analytical solutions are advantageous since they yield a good insight into the significance of the different parameters characterizing the transfer process i.e. diffusivity and water uptake coefficient. Moreover, they are useful as benchmark for numerical models, or for inverse determination of material properties as e.g. the diffusivity itself.

This study aims at addressing the above issue by solving, via an analytical procedure, the moisture transfer problem characterizing the water absorption in capillary active materials. The challenge is given by the fact that, as observed by several authors (e.g. Refs. [9,10,15]), the water diffusivity strongly depends on the water content, leading to a non-linear diffusion equation. To tackle this problem, the diffusivity is here assumed as a stepwise variable function. Hence, the mathematical problem is solved using an approach proposed by Crank [16,17] and already used in polymer science [18,19]. To the knowledge of the authors, this approach is here applied for the first time to describe the water absorption in capillary active building materials. It is shown that the water uptake can be treated as a free boundary problem, in which the porous body is divided into a nearly dry and a nearly saturated region. The boundary between these two regions moves on inside the sample during the absorption process, and it is determined as the locus at which the diffusivity changes. The proposed model is able to explain the absorption behavior of certain building materials, as proved later on in the paper by comparing the analytical solution with measured data found in the literature.

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https://doi.org/10.1016/j.buildenv.2017.09.018
0360-1323/

Nomenclature

$a; b$	functions Eq. (23) [m]
A	surface $[m^2]$
A_w	water uptake coefficient $[kg/(m^2\sqrt{s})]$
$C_{11}; C_{21}$	coefficients Eq. (18) $[\sqrt{s}/m]$
$C_{12}; C_{22}$	coefficients Eq. (18) $[m^3/m^3]$
D_w	diffusivity $[m^2/s]$
g	function Eq. (22) [−]
m_w	cumulative water uptake [kg]
s	integration variable $[m/\sqrt{s}]$
t	time [s]
T	absolute temperature [K]
u	volumetric water content $[m^3/m^3]$
u^*	limit water content $[m^3/m^3]$
x	coordinate [m]
x^*	water front position [m]
η	viscosity $[kg/(m\ s)]$
λ	Boltzmann variable $[m/\sqrt{s}]$
λ^*	limit value of λ $[m/\sqrt{s}]$
σ	surface tension [N/m]
ρ_w	density $[kg/m^3]$
Ψ	porosity $[m^3/m^3]$

Subscripts

$i = 1; 2$	index
ref	reference
0	initial

2. The water uptake test

As stated above, the water uptake test is a widely employed experiment used e.g. for characterizing the diffusivity and the water uptake coefficient of capillary active building materials. In Fig. 1 a scheme of the test is shown. During the experiment, an initially dry material sample is keeping in contact with liquid water or with a saturated textile at its bottom side. Hence, moisture rises into the material due to capillary suction. The absorption rate can be measured by weighing the sample at different times. It is also possible to determine the moisture distribution e.g. by means of X-ray projection [12], neutron radiography [13] or nuclear magnetic resonance [10]; [11]. From these experimental outcomes, the water uptake coefficient and the water diffusivity can be derived, employing e.g. the techniques proposed by Refs. [4] and [10], respectively.

According to several authors (e.g. Refs. [3,4]), the water uptake coefficient A_w $[kg/(m^2\sqrt{s})]$ is defined as follows:

$$A_w = \frac{m_w(t)}{A\sqrt{t}} \tag{1}$$

Here $m_w(t)$ denotes the cumulative water uptake [kg], t the time [s] and A the bottom surface of the absorbing sample $[m^2]$. Note that, according to experimental evidence [3–8], the coefficient A_w is a constant under the hypothesis that the material is homogeneous at the macroscopic scale and that gravity is negligible in comparison with the capillary forces. This has been observed for several building materials (e.g. calcium silicate and ceramic brick), while other materials such as aerated concrete behave differently, see e.g. Ref. [4].

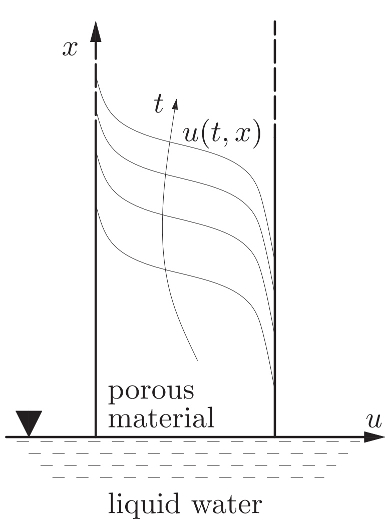

Fig. 1. Scheme of the water uptake test.

The cumulative water uptake appearing in Eq. (1) can be expressed as follows:

$$m_w(t) = A\,\rho_w \Psi \int_0^\infty u(t,x)\mathrm{d}x \tag{2}$$

Here ρ_w denotes the density of liquid water $[kg/m^3]$, Ψ the porosity $[m^3/m^3]$, x the position inside the sample [m] and $u(t,x)$ the volumetric water content $[m^3/m^3]$, varying between 0 (dry material) and 1 (saturated material).

3. Model description

In this section, the mathematical problem describing the above water uptake experiment is introduced and solved via an analytical procedure.

3.1. Transfer equation and boundary conditions

We consider isothermal absorption in a material which can be assumed homogeneous at the macroscopic scale and where gravity is negligible when compared to the capillary forces. Then, according to several authors ([3,10] among others), the one-dimensional capillary moisture transfer is described by the following diffusion equation:

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D_w(u)\frac{\partial u}{\partial x}\right) \tag{3}$$

Here $D_w(u)$ denotes the water diffusivity $[m^2/s]$. The initial and boundary conditions are given by the following equations:

$$u(0,x) = 0 \tag{4}$$

$$u(t,0) = 1 \tag{5}$$

$$\lim_{x\to\infty} u(t,x) = 0 \tag{6}$$

while the diffusivity $D_w(u)$ is here approximated with a step function as follows: