Composite Structures 171 (2017) 528–537

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Free vibration of composite sandwich plates and cylindrical shells

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ARTICLE INFO

Article history:
Received 27 September 2016
Revised 26 December 2016
Accepted 9 March 2017
Available online 19 March 2017

Keywords:
Free vibration
Natural frequency
Laminated sandwich plates
Laminated sandwich shells

ABSTRACT

Sandwich plates and cylindrical shells composed of two composite laminated faces and an ideally orthotropic elastic core are considered in this paper. Since the natural frequencies of sandwich structures may not be affected by the accuracy of local behaviors, to avoid the complexity involved in the higher-order shear deformation theory and layerwise theory and to supplement the loss of the transverse shear deformation in the classical lamination theory, a modified first-order shear deformation theory was employed to obtain the closed-form solutions of natural frequencies of certain particular problems of sandwich plates and shells such as a rectangular composite sandwich plate with symmetric cross-ply laminates with all edges simply supported. Mathematical formulation extended by the classical methods used in isotropic thin plates was also established to deal with the general cases of sandwich plates and cylindrical shells. Numerical results show that the solutions obtained by the present methods are accurate enough to serve as a quick check for the other numerical solutions.

1. Introduction

Laminated composites consisting of layers of various materials are designable materials which can provide excellent opportunity for light weight and high stiffness. To increase the bending stiffness, composite sandwich plates with composite laminated faces and an ideally orthotropic honeycomb core are usually considered in engineering design. To have a proper analysis of this kind of structures, several different mathematical models have been proposed in the literature such as classical lamination theory [1], first-order shear deformation theory [2,3], higher-order shear deformation theory [4], and layerwise theory [5], etc.

To have an accurate prediction on the mechanical behavior of composite sandwich plates, the higher-order and layerwise theories are usually suggested in the literature [6–10]. However, due to the improvement on the mathematical modeling, their formulation becomes more complicated than the simple classical plate theory, and hence, the closed-form solutions become unobtainable. Unlike the local responses such as interfacial stresses which cannot be predicted correctly by classical plate theory, the natural frequencies belonging to the global responses of sandwich structures may not be affected by the accuracy of the local behaviors. To get simple closed-form solutions which are useful for the quick check of other numerical solutions, the most simple model - classical lamination theory may be considered. However, because the classical lamination theory is an extension of classical plate theory [11] without considering the effects of transverse shear deformation, its prediction may fail even on the global responses for sandwich plates and shells. Thus, the first-order shear deformation theory proposed in [12,13], that assumes transverse shear strains be independent of the variable in thickness direction, was employed to solve the problems of free vibration of composite sandwich plates.

By considering the curvature effects, same approach was also extended to the problems of circular cylindrical shells made by composite laminated sandwiches. The classical methods for the analysis of isotropic thin plates such as Navier's solution, Levy's solution, and Ritz method were successfully extended to the present study. Through this extension, some closed-form solutions were obtained for the specific composite laminated sandwiches under specific boundary conditions, and also some mathematical formulations were established for the general cases of composite laminated sandwich plates and cylindrical shells. Verification of the results is then made by comparison between the solutions obtained by these three methods and/or the solutions obtained by the commercial finite element software ANSYS.

2. Mathematical modeling of composite sandwich cylindrical panels

Consider a sandwich cylindrical panel with composite laminated faces and an ideally orthotropic core (Fig. 1). To simplify

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http://dx.doi.org/10.1016/j.compstruct.2017.03.042
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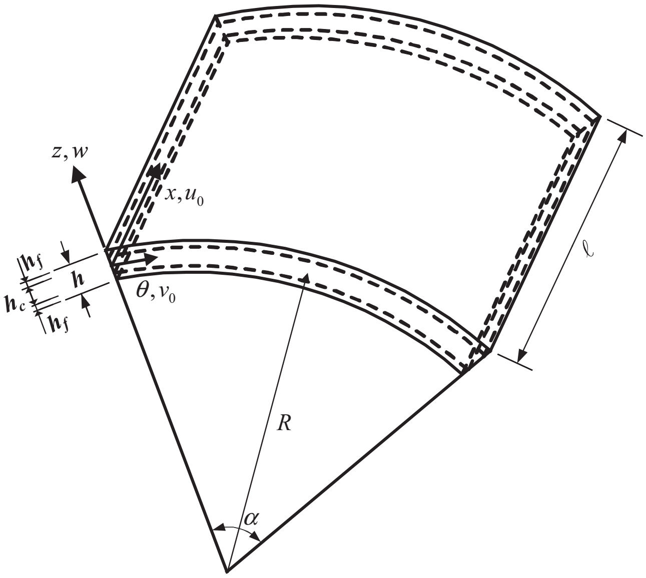

Fig. 1. Geometry and nomenclature for a composite sandwich cylindrical panel.

the analysis, the following assumptions are usually made for sandwiches. (1) The faces are relatively thinner than the depth of the core. (2) The faces take almost all the in-plane loadings and bending moments, while the core undergoes only transverse shear and normal forces. (3) The face-to-core bond ensures that any displacement in the core adjacent to the faces is reproduced exactly in faces, and vice versa. According to these assumptions, a mathematical model for the buckling analysis of composite sandwich plates was established by Hwu and Hu [12]. Based upon Hwu and Hu's model, the kinematic relations, the constitutive laws, and the equations of motion for the vibration analysis of composite sandwich cylindrical panels can be expressed as follows.

(i) Kinematic relations

Let x, θ, and z be the axial, circumferential and outward normal directions of the cylindrical panel (see Fig. 1). The displacements u, v, and w in the directions of x, θ, and z, at time t can be expressed as

$$\begin{aligned} u(x,\theta,z,t) &= u_0(x,\theta,t) + z\beta_x(x,\theta,t),\\ v(x,\theta,z,t) &= v_0(x,\theta,t) + z\beta_\theta(x,\theta,t),\\ w(x,\theta,z,t) &= w_0(x,\theta,t), \end{aligned} \tag{1}$$

where u_0, v_0, and w_0, are the mid-plane displacements in the directions of x, θ, and z; β_x and β_θ are the rotation angles with respect to the x and y directions, and are related to the transverse shear strains $\gamma_{xz}, \gamma_{\theta z}$ by

$$\beta_x = \gamma_{xz} - \frac{\partial w}{\partial x}, \quad \beta_\theta = \gamma_{\theta z} - \frac{\partial w}{R\partial\theta} + \frac{v_0}{R}, \tag{2}$$

in which R is the radius of cylindrical panel. If small deformations are considered, the strains can be written in terms of the mid-surface displacements as follows

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} = \varepsilon_x^0 + zk_x, \quad \varepsilon_\theta = \frac{\partial v}{R\partial\theta} = \varepsilon_\theta^0 + zk_\theta,\\ \gamma_{x\theta} &= \frac{\partial u}{R\partial\theta} + \frac{\partial v}{\partial x} = \gamma_{x\theta}^0 + zk_{x\theta}, \end{aligned} \tag{3a}$$

where

$$\begin{aligned} &\varepsilon_x^0 = \tfrac{\partial u_0}{\partial x}, \quad \varepsilon_\theta^0 = \tfrac{\partial v_0}{R\partial\theta} + \tfrac{w}{R}, \quad \gamma_{x\theta}^0 = \tfrac{\partial u_0}{R\partial\theta} + \tfrac{\partial v_0}{\partial x},\\ &k_x = \tfrac{\partial\beta_x}{\partial x}, \quad k_\theta = \tfrac{\partial\beta_\theta}{R\partial\theta}, \quad k_{x\theta} = \tfrac{\partial\beta_x}{R\partial\theta} + \tfrac{\partial\beta_\theta}{\partial x} \end{aligned} \tag{3b}$$

(ii) Constitutive laws

The stress resultants $(N_x, N_\theta, N_{x\theta})$, bending moments $(M_x, M_\theta, M_{x\theta})$ and transverse shear forces (Q_x, Q_θ) are related to the mid-plane strains $(\varepsilon_x^0, \varepsilon_\theta^0, \gamma_{x\theta}^0)$, curvatures $(\kappa_x, \kappa_\theta, \kappa_{x\theta})$ and transverse shear strains $(\gamma_{xz}, \gamma_{\theta z})$ by

$$\begin{aligned} &\begin{Bmatrix} N_x\\ N_\theta\\ N_{x\theta}\\ M_x\\ M_\theta\\ M_{x\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16}\\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26}\\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66}\\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16}\\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26}\\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0\\ \varepsilon_\theta^0\\ \gamma_{x\theta}^0\\ k_x\\ k_\theta\\ k_{x\theta} \end{Bmatrix},\\ &\begin{Bmatrix} Q_x\\ Q_\theta \end{Bmatrix} = \begin{Bmatrix} A_{55}\gamma_{xz}\\ A_{44}\gamma_{\theta z} \end{Bmatrix}, \end{aligned} \tag{4}$$

where $A_{ij}, B_{ij}, D_{ij}, i,j = 1,2,6,$ and (A_{44}, A_{55}) are the extensional, coupling, bending stiffnesses and transverse shear stiffnesses defined as

$$\begin{aligned} A_{ij} &= \sum_{k=1}^{n} (\bar{Q}_{ij})_k (z_k - z_{k-1}),\\ B_{ij} &= \tfrac{1}{2}\sum_{k=1}^{n} (\bar{Q}_{ij})_k (z_k^2 - z_{k-1}^2),\\ D_{ij} &= \tfrac{1}{3}\sum_{k=1}^{n} (\bar{Q}_{ij})_k (z_k^3 - z_{k-1}^3), \quad i,j = 1,2,6,\\ A_{44} &= \alpha h_c G_{\theta z}, \quad A_{55} = \alpha h_c G_{xz}. \end{aligned} \tag{5}$$

In Eq. (5), $(\bar{Q}_{ij})_k$ is the transformed reduced stiffness of the kth lamina (lamina in both upper and lower faces, not including the core); z_k denotes the z coordinate of the top surface of the kth lamina (see Fig. 2); h_c is the core thickness; α is the shear coefficient

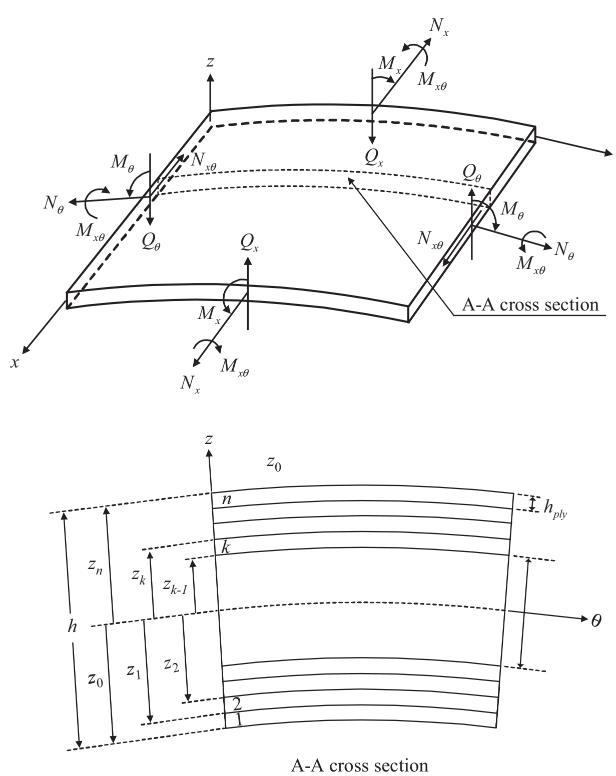

Fig. 2. Stress resultants and bending moments on a panel element.