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Buckling and free vibration of shear-flexible sandwich beams using a couple-stress-based finite element

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ABSTRACT

In this work, we create a framework for linear buckling and free vibration analyses of sandwich beams using a microstructure-dependent Timoshenko beam model founded on the modified couple-stress theory. The stiffness parameters of a structural web-core sandwich panel are determined by unit cell analysis. An extension to homogeneous cores is also carried out. By employing the exact general solution to the governing equations of the beam, an accurate approximate finite element stiffness matrix is formulated. Furthermore, the static shape functions are used to derive consistent linear geometric stiffness and mass matrices. A convergence study shows that the approximate finite element has good accuracy although the hyperbolic terms of the exact general solution have been expanded into only relatively low-order polynomial series. Results from examples show that the microstructure-dependent beam can predict critical buckling loads and natural frequencies with very good accuracy when compared to more sophisticated finite element models. Unlike the classical Timoshenko beam model, the microstructure-dependent model yields accurate results also when the sandwich assembly is transversely flexible and the bending stiffness of the faces non-negligible.

1. Introduction

Sandwich panels are lightweight assemblies composed of two faces and a low-density core. They offer weight-efficiency and can integrate supplementary functions such as noise damping, fire and impact resistance [9]. Models for beam and plate sandwich structures can be divided into two major classes: layerwise models that accurately describe the 3D stress field at ply level, and computationally more efficient equivalent single-layer (ESL) models that are optimal for determining the global response of a component [34,36,7,15]. First-order shear deformation ESL beams and plate models for sandwich members, which are of particular interest in this study, are known to work well in most practical applications [24,1,45]. However, the first-order theories have been found to perform unsatisfactorily in shear-flexible panel applications, where non-negligible shear forces and local bending moments are induced at the faces, stiffening the response locally. To this end, higher-order ESL models have been developed for the study of sandwich panels where local effects play a role (e.g. [11,26]). More recently, microstructure-dependent first-order models, which largely retain the structure of conventional models, have emerged as an alternative to the theoretically involved higher-order ESL models [38,39]. In this paper, we formulate and apply first-order microstructure-dependent ESL beam finite elements to capture local effects that take place in sandwich panels.

Microstructure-dependent continuum mechanics theories, in particular the couple-stress theories [32,42,21], have been revived in recent years mainly due to length scale observations in small scale structures [22,31,23]. In this context, the modified couple stress theory of Yang et al. [44] has become popular as only a single non-classical parameter is introduced into the constitutive relations. Following the modified couple-stress framework, Ma et al. [28], Asghari et al. [3], Asghari et al. [4] and Reddy [35] derived microstructure-dependent Timoshenko beam models for shear deformable structures with length-scale effects. Approximate finite element (FE) beam models were then proposed by Arbind and Reddy [2] and by Kahrobaiyan et al. [18] and recently different FE beam models have been compared by Dehrouyeh-Semnani and Bahrami [10]. Karttunen et al. [20] proposed a nodally-exact element for the microstructure-dependent first-order Timoshenko beam. In this paper, based on their exact general solution, we derive a simplified yet accurate beam finite element, including the linear geometric stiffness and mass matrices. This way global buckling and free vibration analyses of transversely flexible sandwich structures including local effects can be carried out.

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Global buckling and vibrations are important limit states in sandwich structural design. Sandwich structures are often slender, have relatively low mass and applications in vibration-prone structures are extensive. Simplified methods for buckling of sandwich beams are numerous [1,16,5], and higher-order [11] and exact elasticity [19] models have also been proposed. The free vibration problem has usually been addressed with higher-order theories in the single-layer framework [17,12]. The methods available provide trade-offs between accuracy, computational cost and ease in treating variables and boundary conditions. A microstructure-dependent Timoshenko finite element based on Ma et al. [28], Reddy [35], and Karttunen et al. [20] can simplify the buckling and vibration analysis of sandwich beams. Parameter determination is straightforward as only one non-classical parameter which is shown to be related to the local bending of the two sandwich faces is required. The microstructure-dependent beam model is compatible with traditional kinematical quantities, and the beam finite element provides a convenient way for treating different loading and boundary conditions and integrating design models for structural optimization.

The objective of this paper, in more detail, is to create a framework for global buckling and free vibrations of sandwich beams with transversely flexible cores using a microstructure-dependent Timoshenko beam model. An approximate finite element is developed based on the exact element of Karttunen et al. [20] and consistent geometric stiffness and mass matrices formulated. In Section 2, the governing couple-stress Timoshenko beam equations are summarized according to Ma et al. [28] and Reddy [35]. In Section 3, the beam parameters are determined by infinitesimal unit cell analysis for a discrete web-core sandwich geometry and then the results are generalized to homogeneous cores. The exact finite element model of Karttunen et al. [20] is employed in Section 4 to obtain accurate approximate polynomial shape functions, and consistent geometric stiffness and mass matrices are derived for the microstructure-dependent beam. Section 5 presents four numerical case studies. The numerical convergence of the developed approximate beam element is assessed first to better understand the shape function requirements of couple-stress beam models. Then the sandwich beam model and the finite element are validated for cylindrical buckling and free vibrations of honeycomb, web-core and low-density foam core sandwich panels. In Section 6, the main conclusions from the study are drawn and future work briefly discussed.

2. Modified couple-stress Timoshenko beam

Consider a microstructure-dependent Timoshenko beam model (Fig. 1) of length l_e and rectangular cross-section of thickness t and height h [28,35]. The displacement field of the beam is given by

$$U_x(x,z) = z\phi(x), \quad U_z(x,z) = w(x) \tag{1}$$

where ϕ and w are the cross-sectional rotation and the transverse deflection, respectively. The strain field has the following non-zero components

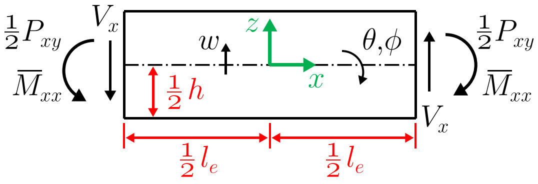

Fig. 1. Microstructure-dependent beam with dimensions, stress resultants and kinematical quantities depicted. $\bar{M}_{xx} = M_{xx} + 1/2P_{xy}$ and $V_x = Q_x + 1/2\,\partial P_{xy}/\partial x$.

$$\begin{aligned} \varepsilon_{xx} &= z\tfrac{\partial\phi}{\partial x} \\ \gamma_{xz} &= \phi + \tfrac{\partial w}{\partial x} \\ \chi_{xy} &= \tfrac{1}{4}\left(\tfrac{\partial\phi}{\partial x} - \tfrac{\partial^2 w}{\partial x^2}\right) \end{aligned} \tag{2}$$

where ε_{xx} and γ_{xz} are the axial normal strain and the transverse shear strain, respectively, and χ_{xy} is the curvature. The axial normal stress, transverse shear stress and the couple-stress are obtained from the constitutive relations

$$\sigma_{xx} = E(z)\varepsilon_{xx}, \quad \tau_{xz} = G(z)\gamma_{xz}, \quad m_{xy} = 2G(z)l^2\chi_{xy} \tag{3}$$

The equilibrium equations of the beam under an external load $q(x)$ are (see [35])

$$\frac{\partial Q_x}{\partial x} + \frac{1}{2}\frac{\partial^2 P_{xy}}{\partial x^2} = -q(x) \tag{4}$$

$$Q_x - \frac{\partial M_{xx}}{\partial x} - \frac{1}{2}\frac{\partial P_{xy}}{\partial x} = 0 \tag{5}$$

In this work, the microstructure-dependent beam is taken to represent a sandwich panel with two face plates and a low-density core. The stress resultants in Eqs. (4) and (5) are defined as

$$\begin{aligned} M_{xx} &= \textstyle\int_A \sigma_{xx} z dA = D_x\tfrac{\partial\phi}{\partial x} \\ Q_{xz} &= \textstyle\int_A \tau_{xz} dA = D_Q\left(\phi + \tfrac{\partial w}{\partial x}\right) \\ P_{xy} &= \textstyle\int_A m_{xy} dA = \tfrac{S_{xy}}{2}\left(\tfrac{\partial\phi}{\partial x} - \tfrac{\partial^2 w}{\partial x^2}\right) \end{aligned} \tag{6}$$

where D_{xx} and D_Q are the bending and shear stiffness, respectively, and S_{xy} is the couple-stress related stiffness of the beam. The boundary conditions are determined by specifying one element in each of the following pairs at the beam ends

$$\begin{aligned} &Q_x + \tfrac{1}{2}\tfrac{\partial P_{xy}}{\partial x} \text{ or } w \\ &M_{xx} + \tfrac{1}{2}P_{xy} \text{ or } \phi \\ &\tfrac{1}{2}P_{xy} \text{ or } \theta \equiv -\tfrac{\partial w}{\partial x} \end{aligned} \tag{7}$$

3. Determination of stiffness parameters D_Q, D_{xx}, and S_{xy}

In the following, we determine the single-layer couple-stress beam stiffness parameters of a web-core sandwich panel and extend the derivation to transversely flexible homogeneous cores. An infinitesimal unit cell that is representative of the whole panel on average is studied. The unit cell analysis is facilitated by the fact that the local bending of the face plates can be accounted for by the couple-stress resultant P_{xy}, whereas the bending moment M_x is associated with the bending stiffness which is due to the membrane action of the face plates. Except for the addition of the couple-stress resultant P_{xy} to the analysis, the approach is similar to that carried out to obtain the stiffness parameters for conventional first-order beam and plate theories [25,13,14,27,30,43,6].

3.1. Stiffness parameters of a web-core beam

A repetitive unit cell of a periodic web-core beam is shown in Fig. 2. A half of the unit cell is considered to represent an infinitesimal element of length $s \approx dx$. The distance between the centerlines of the faces is d. Bending rigidities of the face and web plates are EI_f and EI_w, respectively, and EA_f is the axial rigidity of a face.