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Bending, buckling and vibration of axially functionally graded beams based on nonlocal strain gradient theory

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ABSTRACT

A size-dependent inhomogeneous beam model, which accounts for the through-length power-law variation of a two-constituent axially functionally graded (FG) material, is deduced in the framework of the nonlocal strain gradient theory and the Euler–Bernoulli beam theory. By employing the Hamilton principle, the equations of motion and boundary conditions for size-dependent axially FG beams are deduced. A material length scale parameter and a nonlocal parameter are introduced in the axially FG beam model to consider the significance of strain gradient stress field and nonlocal elastic stress field, respectively. The bending, buckling and vibration problems of axially FG beams are solved by a generalized differential quadrature method. The influences of power-law variation and size-dependent parameters on the bending, buckling and vibration behaviors of axially FG beams are investigated. The mechanical behaviors can be affected by the through-length grading of the FG material and therefore may be controlled by choosing appropriate values of the power-law index. When considering concentrated and uniformly distributed loads, the maximum deflection decreases with increasing length scale parameter. The axially FG beam may exert a stiffness-softening effect or a stiffness-hardening effect on the critical buckling force and the natural frequencies depending on the values of the two size-dependent parameters.

1. Introduction

Inhomogeneous materials are ubiquitous in engineering, such as spacecraft heat shields, heat exchanger tubes, biomedical implants, etc. Müller [1] experimentally investigated a kind of segmented inhomogeneous materials, whose properties varying along length direction of the beam (axially functionally graded (FG) beam), used in micro sensors. FG materials can be characterized by the variation in composite structures gradually over volume and can be considered as continuous materials variant in one or more dimensions, resulting in corresponding changes in their material properties. The FG materials gradually change their micro-structures from one material to another with a certain gradient and can be designed for specific functions and applications. Many methods have been used to fabricate the functionally graded materials by using particulate processing, preform processing, layer processing and melt processing. The beam made of FG materials, with properties varying through the length direction, can meet a specific purpose under inhomogeneous conditions such as under a graded temperature field, suffering non-uniformly distributed load, etc. The potential usage of axially functionally graded micro/nanosensors and micro/nanoactuators is also under investigation. Although there exist many papers focus on functionally graded beams in thickness direction, the research devoted to axially functionally graded beams is limited. The small-scaled FG materials possibly offer novel great designs and applications in nano-electro-mechanical (NEMS) and micro-electro-mechanical systems (MEMS), where the size-dependent effect is worthy studying and understanding. It was shown by many experimental studies that some significant size-dependent effects can be observed on the mechanical and physical properties of nano/micro-scaled systems. It means that when the external dimension or volume of structures changes into micro-scale, the size effect become significant and must be considered. These size-dependence effects are related to the size of micro unit cells (the so-called material internal lengths), including a molecule of polymer materials, a crystallite of polycrystal raw materials, or a grain of granular materials [2]. That is, the studies of size-dependence effects on the mechanical behaviors should be related to both internal lengths and external dimensions. Thus, the studies of the size-dependent effects on the bending, buckling and vibration behaviors of axially functionally graded beams are always of fundamental significance.

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The nonlocal elasticity theory [3] states that the stress field at a reference point is assumed to depend not only on the strain at the reference point but also on the strains at all other points in the whole body. The nonlocal continuum theory may reasonably explain size-dependent phenomena of mechanical behaviors of small-scaled structures [4–7]. Many nonlocal continuum models [8–20] are developed to study the size-dependent effects on the static and dynamic behaviors of small-scaled structures. More recently, Nejad et al. [21] investigated the buckling behavior of arbitrary two-directional FG nanobeams based on the Euler–Bernoulli beam theory and nonlocal elasticity theory. These works showed that the nonlocal effects potentially play a important role in studying the size-dependent effects on the static and dynamic behaviors of small-scaled FG beams and stiffness-softening effects were reported for these nonlocal elasticity models. Aifantis [22] proposed the simplest form of the strain gradient theory with only one additional length scale parameter required. Metrikine and Askes [23] states that there are three equivalent approaches (phenomenological, statistical and continualization approaches) to formulate the size-dependent constitutive relation. Iliopoulos et al. [24] determined the length scale parameter in terms of lattice model and experimentally verified in terms of wave properties. In this case, there are totally four material parameters (including the Lame constants and length scale parameters for isotropic materials). Couple stress theory may be a special case of Mindlin's strain gradient theory [25] with neglecting the symmetric part of the gradient of strain. Yang et al. [26] proposed the modified couple stress theory including only one additional parameter and considering a symmetric couple stress tensor. Reddy [10] studied the bending, vibration and buckling problems of FG beams by using Euler–Bernoulli and Timoshenko beams models as well as modified couple stress theory. Beni [27,28] studied the static and dynamic behaviors of size-dependent Euler–Bernoulli and Timoshenko beams. Akgöz and Civalek [29,30] developed shear deformation beam models of micro-scaled FG beams based on the modified gradient elasticity theories. Khorshidi et al. [31] investigated the post-buckling behavior of FG nanobeams based on modified couple stress theory by consider general beam theory. Taati [32] performed the buckling analysis of size-dependent FG plates by using the modified couple stress theory. Shojaeian and Beni [33] studied the buckling of FG slender beams based on modified couple stress theory. Shojaeian and Zeighampour [34] investigated the size dependent behavior of beam-type micro/nano electromechanical systems made of functionally graded sandwich materials subjected to intermolecular Casimir forces and accounting for electrostatic actuation effects based on the modified couple stress theory. Zeighampoura and Beni [35] analyzed a circular strain gradient axially functionally graded beam with various radius along the length direction. Shafiei et al. [36] studied the transverse vibration of a rotary tapered axially FG microbeam based on the modified couple stress theory in the form of true spatial variation. Shafiei et al. [37] performed the size dependent vibration behavior of a rotating non-uniform FG Timoshenko and Euler–Bernoulli microbeam by using the modified couple stress theory. Reddy et al. [38] developed some displacement finite element models for the nonlinear analysis of the classical and first-order FG plate based on the modified couple stress theory. Furthermore, a lot of models [39–50] have been recently developed to study the static and dynamic behavior of FG beams and plates in the framework of the modified gradient elasticity theory. Beni and co-authors [51–55] studied the mechanical properties of FG cylindrical shells based on modified couple stress theory. Stiffness enhancement effects were reported for these gradient elasticity models.

From the literature discussed above, it is clear that the nonlocal elasticity models and the strain gradient models are two different models describing two size-dependent effects on the mechanical characteristics of small-scaled materials. The nonlocal strain gradient model was firstly developed to assess the two length scales effects on mechanical behaviors by combining the local and nonlocal curvatures in the constitutive relation [56]. An excellent matching of the dispersive relation of between the Born-Kármán model of lattice dynamics and the nonlocal strain gradient model can be found for truss structures [57]. The nonlocal strain gradient models have been developed to study the size-dependent effects on the waves propagating, vibration, bending and buckling behaviors of small-scaled structures [5,6,24,58–64]. Furthermore, the nonlocal elasticity models and the strain gradient models can be cast into a single constitutive equation with the view to removing stress singularities (see, e.g., [65,66]). Lim et al. [67] cast these nonlocal strain gradient models into a single size-dependent theory in the context of thermodynamics framework. The nonlocal strain gradient theory states that the stress of small-scaled materials accounts for not only nonlocal stress field [3], but also accounts for the pure strain gradient stress field [22]. The nonlocal strain gradient model has two independent small length-scale parameters (namely, the material length scale parameter and the nonlocal parameter). It has been reported that the wave dispersive predicted by nonlocal strain gradient models can agree very well with that obtained by molecular dynamics simulations for beam-type structures [5,6]. The nonlocal strain gradient theory has been used to assess the linear and nonlinear size-dependent longitudinal vibration behaviors of carbon nanotubes [68,69]. Based on the nonlocal strain gradient theory, Li and Hu [70] studied the size-dependent effect on the buckling of nonlinear beams, and Li et al. [71] investigated the size-dependent effects on the critical flow velocity of fluid-conveying microtubes. These works showed that the stiffness-softening or stiffness-enhancement effects can be observed, depending on the values of the nonlocal parameter and the material length scale parameter. Farajpour et al. [72] studied the vibration behavior of piezoelectric nanofilm-based electromechanical sensors based on the non-local strain gradient theory. Farajpour et al. [73] developed a nonlocal strain gradient plate model to buckling of orthotropic nanoplates in thermal environment. Reddy [10] pointed out that a detail examination of the size-dependent effects of the material length scale parameter and the nonlocal parameter on the mechanical behaviors of FG beams is a topic worth exploring elsewhere. Li et al. [74–76] investigated the size-dependent effects on the bending, buckling, and free vibration behaviors of nonlocal strain gradient beams made of through-thickness FG material. Şimşek [77] studied the size-dependent nonlinear free vibration behaviors of a through-thickness FG nanobeam using the nonlocal strain gradient theory. Ebrahimi and Barati [78,79] studied the size-dependence effects on wave and vibration characteristics of through-thickness FG nanobeams based on nonlocal strain gradient theory and the higher-order shear beam theories. Yang et al. [80] performed a nonlinear dynamic analysis of microbeams made of FG material reinforced by carbon nanotubes based on the nonlocal strain gradient theory. However, most of these researches are related to homogeneous beams or through-thickness FG beams. Studies for the size-dependent mechanical behaviors of axially FG beams are in desperate need.

Under the motivation of these facts, this study will be focused on the following aspects:

(i) A unified size-dependent model (i.e., a nonlocal strain gradient beam model) will be deduced to account for the nonlocal and strain gradient effects. Thus, both stiffness softening and enhancement effects will be taken into account. Besides, in the beam model, the axially functionally graded material properties will also be considered to obey the power-law grading along the length direction of the beam.