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A new Fourier-related double scale analysis for wrinkling analysis of thin films on compliant substrates

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ABSTRACT

In this paper, a new Fourier-related double scale approach is presented to study the wrinkling of thin films on compliant substrates. By using the method of Fourier series with slowly variable coefficients, the 1D microscopic model proposed by Yang et al. (2015) is transformed into a 1D macroscopic film/substrate model whose mesh size is independent on the wrinkling wavelength. Numerical tests prove that the new model improves computational efficiency significantly with accurate results, especially when dealing with wrinkling phenomena with vast wavenumbers. Besides, we propose a strategy to efficiently trace the wrinkling pattern corresponding to the lowest critical load by accounting for several harmonics of Fourier series in this new model. The established nonlinear system is solved by the Asymptotic Numerical Method (ANM), which has advantages of efficiency and reliability for stability analyses.

1. Introduction

System of a stiff layer resting on soft substrate exists widely in nature (human skin [2]) and engineering fields (intelligent materials, biomedical field [3]). When subjected to in-plane compression or exposed to the reduction of temperature field [4], the mismatch of mechanical properties or coefficients of thermal expansion of layers may lead to wrinkles. The wrinkles may pose a limit on the performance of materials or structures [5] and are always thought to be avoided, but nowadays may find some applications such as assembly of materials [6], measuring the mechanical properties of materials in modern metrology [7]. For these reasons, it is quite necessary to characterize the wrinkling of the system in an accurate and efficient way.

As early as forty years ago, the stability analysis of multilayered materials was investigated by Allen [5] in the framework of sandwich panel designs in airplanes. From then on, much of theoretical work on substrate-bonded films has been proposed on the basis of linear perturbation analysis (see Niu and Talreja [8]), which focused on determining the critical membrane force and wavelength of wrinkles. Recently, some nonlinear analyses were performed to further investigate and comprehend wrinkle characteristics in film/substrate systems. Using energy calculation, Chen and Hutchinson [9] showed that the herringbone mode constitutes a minimum energy configuration among computed modes for a film under equal biaxial compression. Huang et al. [10] modeled the substrate as an array of spring dashpots and developed a spectral method to inspect the evolution of wrinkling configurations, whose results showed that wrinkles can evolve into several patterns depending on the anisotropy of membrane strain. As an extension work, Huang et al. [11] extended their original spring model to a model representing the substrate by three-dimensional elastic field and investigated the influence of the Young's modulus, the thickness of substrate on the amplitude and wavelength of the sinusoida wrinkles. Through experiments and analytical solutions, Jiang et al. [12] found the width effect on wrinkles of film/substrate that the amplitude and wavelength of wrinkles increase with film width. Unlike the results obtained by the small deformation theory [9–12], Song et al. [13] considered finite strain and nonlinear constitutive law in the substrate, and found that the buckling wavelength does not keep constant as in [9–11] but relates to the large prestrain. To model the film/substrate system more efficiently, Yang et al. [1] proposed a high order model based on the concept of the Carrera's Unified Formulation (CUF) [14,15], and results verified that the CUF model yielded accurate results with low computational cost. Similar methods based on such enriched kinematical functions have been widely used in the mechanical analysis of composite plates [16–19].

In the above film/substrate systems, instability patterns are usually nearly periodic in spatial. Based on an efficient multiscale approach established by Damil and Potier-Ferry [20,21] that exploits such periodic nature and the effective CUF model of Yang

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et al. [1], a Fourier-related one-dimensional model is developed to study the wrinkling phenomena in thin stiff films on compliant substrates. In the framework of the multi-scale approach (often referred to as "Fourier series with slowly varying coefficients"), the unknowns to be solved in Yang et al. [1] are expanded by the Fourier series, which leads to the Fourier coefficients as the new unknowns varying much more slowly than the former unknowns (we address the former unknowns as "microscopic" and the latter "macroscopic" in this paper). Usually using rather coarse spatial meshes is sufficient to describe the slowly varying envelops, therefore computational efficiency can be improved significantly [21–23]. This approach is firstly applied to study the instability phenomena in beams on Winkler foundation [20,21] with a most simplified macroscopic model, in which only the zero harmonic (mean field) and the first order harmonics (envelops) are taken into account. Results showed that the macroscopic model has two advantages compared to Landau-Ginzburg technique: (1) not only the bifurcation point but also the post-buckling path can be captured and (2) the coupled global and local instability patterns can be incorporated and characterised, see Liu et al. [22]. In this paper, a similar most simplified macroscopic model is firstly presented to investigate the instability of the film/substrate structures. As like any reduced models, the Fourier models may loss some accuracy in the vicinity of boundary. To handle this issue, Mhada et al. [24] took into account phase change in macroscopic model, and Hu et al. [25] proposed to establish multi-scale model containing microscopic model near the boundary and macroscopic model in the bulk, the two models were bridged by the Arlequin method [26]. Results showed that the multi-scale model treats boundary conditions better. For the above Fourier models, only two harmonics (the first order envelop and the mean field) were taken into account, thus the wrinkling wavelength or wavenumber should be a prior set variable, which may lead to some inconvenience by trying different wavelengths or wavenumbers to find the lowest critical buckling load. In this paper, we further propose another strategy to trace the lowest critical load by deriving a macroscopic model considering several harmonics of the Fourier series. Results show that, independent on preset wavenumber, the macroscopic model can automatically trace the lowest wrinkling pattern. The established nonlinear system for the instability problems of the film/substrate structures shows strong nonlinearity with bifurcation branches, and it is difficult to trace the so-called equilibrium path. Two kinds of widely used methods to solve these nonlinear problems can be the classical predictor-corrector methods (e.g. the Newton-Raphson method and the arc-length method) and the perturbation methods [27]. By coupling a perturbation method with numerical method, Potier-Ferry et al. [28–30] developed an efficient nonlinear solver known as the Asymptotic Numerical Method (ANM), which is effective and robust especially in tracing bifurcation path in instability problems. In this paper, the ANM is used to solve the established nonlinear equations.

This paper is structured as follows. The 1D microscopic model for film/substrate system is reviewed in Section 2. In Section 3, the macroscopic model is deduced by the Fourier series, and the nonlinear equations are solved by the Asymptotic Numerical Method. In Section 4, numerical tests have been carefully investigated to validate the established macroscopic model.

2. Microscopic model

2.1. Kinematics

We consider a two-dimensional elastic stiff film resting on an elastic soft compliant substrate as depicted in Fig. (1). Let x and z be the longitudinal and the transverse coordinates, L and b are the length and the width of the structure, respectively. The thicknesses of the top film and the substrate are, respectively, h_f and h_s. The substrate is ideally divided into a core and a bottom layer of thickness h_c and h_b as in [1]. This division allows to describe the rapid displacement variation along the z direction close to the thin film by high-order kinematic theories and slowly varying kinematics far away from the top film by low-order polynomials, so that less variables are needed to accurately describe the displacement field in the film/substrate structure.

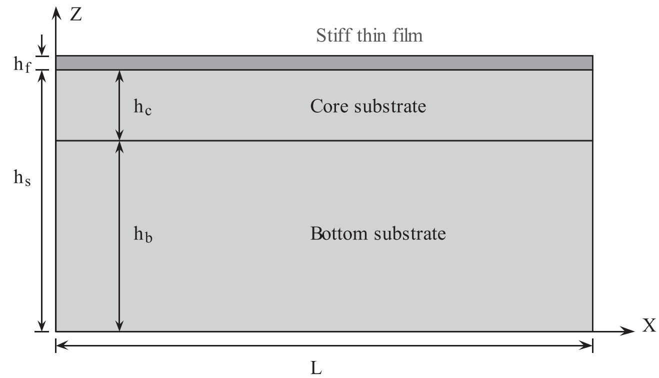

Fig. 1. Sketch of an elastic thin stiff film on a compliant substrate.

As the same kinematic proposed in [1], the thin film is modeled as the Euler–Bernoulli's beam:

$$\text{Film}: \begin{array}{l} \mathscr{U}^f(x,z) = u_0^f(x) - (z - \frac{h_f+2h_s}{2})\mathscr{W}^f_{,x}(x,z) \\ \mathscr{W}^f(x,z) = w^f(x) \end{array} \qquad z \in [h_s, h_s + h_f] \tag{1}$$

where the superscript "f" stands for the film, the notation "$,x$" stands for $\frac{\partial}{\partial x}$, $\mathscr{U}(x,z)$ and $\mathscr{W}(x,z)$ represent the displacements along the $x-$ and $z-$axis, respectively. The two unknown functions u_0^f and w^f are the components of the displacements of the mid-plane of the film. The substrate is modeled as a plane-strain elastic solid:

$$\text{Core}: \begin{array}{l} \mathscr{U}^c(x,z) = F_\tau(z)u_\tau^c(x) \\ \mathscr{W}^c(x,z) = F_\tau(z)w_\tau^c(x) \end{array} \qquad z \in [h_b, h_s] \quad \tau \in [0, n_c] \tag{2}$$

$$\text{Bottom}: \begin{array}{l} \mathscr{U}^b(x,z) = F_\tau(z)u_\tau^b(x) \\ \mathscr{W}^b(x,z) = F_\tau(z)w_\tau^b(x) \end{array} \qquad z \in [0, h_b] \quad \tau \in [0, n_b] \tag{3}$$

where superscripts "c" and "b" stand for the core and bottom layer of substrate, respectively. The $F_\tau(z)$ represents the through-the-thickness approximating function within CUF framework, which generally can be an element of a generic approximation base. Within this work, the Mac Laurin's polynomials z^n are adopted as expansion function. The notations u_τ and w_τ are the unknown displacement functions along the beam axis. According to the Einstein rule, repeated indexes denote summation, thus $\mathscr{U}^c(x,z)$, as an example, can be explicitly written as:

$$\mathscr{U}^c(x,z) = u_0^c + zu_1^c + z^2u_2^c + \cdots + z^nu_n^c \qquad n \in \mathbb{N} \tag{4}$$

As a result, a family of one-dimensional refined beam models can be systematically obtained considering the expansion function order as a free parameter of the formulation, that is, n can be an arbitrary value. In this film/substrate system, the variable n is valued as n_c and n_b for the core and bottom layer of substrate, respectively.

The continuity of the displacement field between the layers film/core and core/bottom is ensured by