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On Bending-Twisting coupled laminates

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ABSTRACT

The definitive list of stacking sequences is presented for *Bending-Twisting* coupled ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$) laminates, with up to 21 plies. This class of laminate arises from the ubiquitous balanced and symmetric design rule, but symmetry is shown to be a sufficient rather than a necessary constraint. Each stacking sequence configuration is derived in symbolic form together with dimensionless parameters from which the extensional and bending stiffness terms are readily calculated for any fibre/matrix system and angle-ply orientation. Expressions for ply orientation dependent lamination parameters are also given, together with graphical representations, which demonstrate the extent of the design space. Pseudo Quasi-Homogeneous ($\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$) laminates are introduced as an important laminate sub-set, since such laminates have concomitant orthotropic properties, i.e. matching orthotropic or isotropic stiffnesses in extension and bending, from which the isolated effects of *Bending-Twisting* coupling can be studied. These coupling effects are quantified for compression buckling of Angle-ply and Quasi-Isotropic laminated plates with simply supported and clamped edges.

1. Introduction

This article is one in a series, investigating unique forms of thermo-mechanical coupling behaviour. These are described collectively in an original article [1], identifying all 24 possible coupling interactions between *Extension*, *Shearing*, *Bending* and *Twisting*.

Here, attention is focussed on the identification of laminated composite materials possessing isolated mechanical *Bending-Twisting* coupling, i.e., with no other coupling present. It complements a previous article on isolated mechanical *Extension-Shearing* coupling [2].

Stacking sequence listings are derived for *Bending-Twisting* coupled laminates with up to 21 plies; or 42 plies if the data is interpreted as the symmetric half of the laminate stacking sequence definition.

Symmetric stacking sequences are ubiquitous in composite laminate design practice, for the simple reason that their use guarantees the laminate will remain flat, or warp free, after high temperature curing. Non-symmetric laminates are commonly associated with, or often (incorrectly) used to describe [4], configurations that warp extensively after high temperature curing. However, non-symmetric stacking sequence configurations will be shown to provide the same thermo-mechanical properties as their symmetric counterpart, but are part of a much larger and generally unexplored design space. Balanced stacking sequences guarantee that *Extension-Shearing* coupling is eliminated by using matching pairs of angle-ply layers [5].

With very few exceptions, composite designs for aircraft construction are currently certified only for balanced and symmetric laminates [6], which may possess fully uncoupled stiffness properties [3] as well as *Bending-Twisting* coupling and therefore this common design rule is by no means the panacea for characterisation of mechanical behaviour.

In this article, laminate stacking sequence configurations are derived in symbolic form together with dimensionless parameters from which the extensional and bending stiffness terms are readily calculated; solutions are therefore independent of the fibre/matrix system and angle-ply orientation. Expressions relating the dimensionless parameters to the well-known ply orientation dependent lamination parameters are also given, together with graphical representations of feasible domains for a range of ply number groupings (n) including typical fuselage skin thicknesses, i.e., with between (n =) 12 and 16 plies.

Quasi-Isotropic laminates are presented as an important sub-set of *Bending-Twisting* coupled laminates. Pseudo Quasi-Homogeneous laminates are also introduced, since such laminates have concomitant orthotropic properties, i.e. matching orthotropic stiffness in extension and bending, from which the isolated effects of *Bending-Twisting* can be studied; especially where concomitant properties are isotropic in nature. Quasi-Homogeneous designs

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Nomenclature

A, A_{ij}	extensional stiffness matrix and its elements (i,j = 1, 2, 6)
B, B_{ij}	coupling stiffness matrix and its elements (i,j = 1, 2, 6)
D, D_{ij}	bending stiffness matrix and its elements (i,j = 1, 2, 6)
$E_{1,2}$, G_{12}	in-plane Young's moduli and shear modulus
H	laminate thickness (= number of plies, $n \times$ ply thickness, t)
k_x	non-dimensional buckling load factor in compression
M	vector of moment resultants (= $\{M_x, M_y, M_{xy}\}^T$)
N	vector of force resultants (= $\{N_x, N_y, N_{xy}\}^T$)
n	number of plies in laminate stacking sequence
Q_{ij}	reduced stiffness (i,j = 1, 2, 6)
Q'_{ij}	transformed reduced stiffness (i,j = 1, 2, 6)
t	ply thickness
U_i	laminate invariant (i = 1,2,3,4,5)
x,y,z	principal axes
z_k	layer k interface distance from laminate mid-plane
ε	vector of in-plane strains (= $\{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}^T$)
$\boldsymbol{\kappa}$	vector of curvatures (= $\{\kappa_x, \kappa_y, \kappa_{xy}\}^T$)
λ	buckling half-wave

ν_{ij}	Poisson ratio (i, j = 1, 2)
θ_k	ply orientation for layer k
ξ_{1-2}	lamination parameters for extensional stiffness
ξ_{9-12}	lamination parameters for bending stiffness
ζ	bending stiffness parameter for laminate (= n^3)
$\zeta_\pm$	bending stiffness parameter for angle-ply sub-sequence
ζ_+, ζ_-	bending stiffness parameter for positive/negative angle-ply sub-sequence
$\zeta_\circ, \zeta_\bullet$	bending stiffness parameter for cross-ply sub-sequences
+,−,±	angle plies, used in stacking sequence definition
○,●	cross-plies, used in stacking sequence definition

Matrix sub-scripts

0	All elements zero
F	All elements <u>F</u>inite
I	<u>I</u>sotropic form
S	<u>S</u>pecially orthotropic or *<u>S</u>imple* form

permit ply percentage and buckling strength contours to be mapped onto the same lamination parameter design space, and thus serve to demonstrate the effect on buckling strength of ignoring the presence of *Bending-Twisting* coupling; examples, including Pseudo Quasi-Homogeneous Quasi-Isotropic laminate designs, are given.

Bending-Twisting coupling is generally known to result in weaker compression buckling strength compared to the equivalent fully uncoupled laminate, i.e., with matching stiffness properties, although there is evidence that this continues to be ignored in design practice as well as in the recent literature [7], leading to potentially unsafe design predictions, despite the guidance provided in long standing articles [8] and the availability of approximate closed form solutions with a high degree of accuracy [9]. Some new insights into the relative buckling strength with respect to lamination parameter design spaces are provided, by way of an introduction to an accompanying article [10], which explores in detail the effect of *Bending-Twisting* coupling on compression and shear buckling strength, and is applicable to the data presented here, as well as to data for *Extension-Shearing* and *Bending-Twisting* coupled laminates.

The remainder of this article is arranged as follows. Section 3 provides an overview of mechanical coupling behaviour before details of the derivation of definitive listings of *Bending-Twisting* coupled laminate configurations are presented, including non-dimensional and angle-ply dependent parameters and example calculations. Section 4 provides information on the extent of the feasible design space for *Bending-Twisting* coupled laminates, with comparison to *Simple* laminates, including the dominant forms of sub-sequence symmetries. Lamination parameters are also introduced to allow the available design space to be visually interrogated. The use of ply percentage mapping, as an approach to design for bending stiffness, is also discussed in the context of Quasi-Homogenous and Pseudo Quasi-Homogeneous laminates. Section 5 describes the association between ply percentages and compression buckling strength, through a similar mapping procedure. Classical Garland curves are then presented in a form that permits an assessment of the bounds on the buckling strength of *Bending-Twisting* coupled laminates subject to compression load. Finally, a note on laminate selection is given in Section 6 before conclusions are drawn in Section 7.

2. Mechanical coupling

Laminated composite materials are characterized in terms of their response to mechanical (and/or thermal) loading, which is associated with a description of the coupling behaviour, unique to this type of material, i.e. coupling between in-plane (i.e. extension or membrane) and out-of-plane (i.e. bending or flexure) responses when $B_{ij} \neq 0$ in Eq. (1), coupling between in-plane shearing and extension when $A_{16} = A_{26} \neq 0$, and coupling between bending and twisting when $D_{16} = D_{26} \neq 0$.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ & A_{22} & A_{26} \\ \text{Sym} & & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ \text{Sym} & & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$
$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ & B_{22} & B_{26} \\ \text{Sym} & & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ & D_{22} & D_{26} \\ \text{Sym} & & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{1}$$

Whilst Eq. (1) describes the well-known **ABD** relation from classical laminate plate theory, it is more often expressed using compact notation:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \end{Bmatrix} \tag{2}$$

The coupling behaviour, which is dependent on the form of the elements in each of the extensional [**A**], coupling [**B**] and bending [**D**] stiffness matrices, is now described by an extended subscript notation, defined previously by the Engineering Sciences Data Unit, or ESDU [5] and subsequently augmented for the purposes of this series of articles. Hence, laminates with coupling between bending and twisting, are referred to by the designation $\mathbf{A}_S\mathbf{B}_0\mathbf{D}_F$, signifying that the elements of the extensional stiffness matrix [**A**] are simple or specially orthotropic in nature, i.e.:

$$\begin{bmatrix} A_{11} & A_{12} & 0 \\ & A_{22} & 0 \\ Sym & & A_{66} \end{bmatrix} \tag{3}$$

the coupling matrix [**B**] is null, whilst all elements of the bending stiffness matrix [**D**] are finite, i.e.: