Composite Structures 155 (2016) 223–244

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

A triangular finite element using Laplace transform for viscoelastic laminated composite plates based on efficient higher-order zigzag theory

Sy-Ngoc Nguyen [a], Jaehun Lee [b], Maenghyo Cho [a,*]

[a] *Department of Mechanical and Aerospace Engineering, Seoul National University, Seoul, 08826, Republic of Korea*
[b] *School of Mechanical Engineering, Kyungnam University, Changwon, 51767, Republic of Korea*

ARTICLE INFO

Article history:
Received 13 April 2016
Revised 19 July 2016
Accepted 20 July 2016
Available online 22 July 2016

Keywords:
Composite laminates
Zig-zag plate theory
Viscoelasticity
Laplace transform
Finite element analysis

ABSTRACT

To predict the time-dependent behaviors of viscoelastic laminated composites, a three-node multilayered plate element is developed based on the efficient higher-order plate theory (EHOPT), which was originally proposed by Cho and Parmerter. With the help of the Laplace transform, the integral form of the constitutive equation in the time domain is reduced to an algebraic equation in the Laplace domain. Thus, the structures and advantages of the EHOPT can be preserved in viscoelastic laminated composites in the Laplace domain. Since the time dimension is transformed to Laplace domain, the finite element discretization is only used in the spatial domain. A nonconforming three-node triangular element is employed to implement the viscoelastic EHOPT for finite element analysis. To pass the proper bending and shear patch tests in arbitrary mesh configurations, the modified shape function developed by Specht is applied and converted into Laplace domain. Therefore, the final numerical results, which is obtained by using inverse Laplace techniques, always converge to the corresponding analytical solutions. In order to verify the efficiency and accuracy of the present study, some numerical examples for long-term creep and relaxation processed are performed. The present viscoelastic finite element of composite laminates provides a powerful tool to accurately investigate the responses of the viscoelastic and time-dependent mechanical behaviors of composite laminates.

1. Introduction

Composite materials have viscoelastic characteristics that depend on either matrices or a combination of reinforcements and matrices. In particular, composite laminates made of carbon fibers and epoxy matrices show anisotropic viscoelastic behaviors. Complex time-dependent viscoelastic behavior is a typical mechanical behavior of composite structures. Because the responses of polymer composite structures significantly depend on the applied loading and environmental conditions, the viscoelastic analysis is essential for the accurate prediction of the time-dependent response of composite structures. However, the analysis of viscoelastic materials is much more complicated than that of an elastic one since the constitutive relation is expressed in a time-dependent integral form. Consequently, the computational cost for viscoelastic composite analysis is extremely expensive. Therefore, an accurate and efficient method for the analysis of viscoelastic composite structures is needed.

Various refined plate theories have been developed for the analysis of elastic composite laminates by considering the effect of transverse shear deformation, which is neglected in the classical laminated plate theory (CLPT), or assumed as a constant in the first-order shear deformation theory (FSDT) [1–3]. Among these theories, the third-order shear deformation theory (TSDT) [4] using a cubic polynomial for the in-plane displacement fields accurately predicts the global behavior of laminates, such as the deflection, natural frequency, and buckling load. Nevertheless, the TSDT does not show good performance in describing local responses, such as the in-plane displacement and stress distributions in the thickness direction of the laminates. Conversely, the layer-wise plate theories [5,6] successfully predict both global and local behaviors by considering layer-dependent variables. However, it makes the number of unknowns in the layer-wise models increase and depends on the number of layers. When composite laminates are used as primary loading structures, hundreds of layers are required to substitute the former metallic structural components. Thus, the

* Corresponding author.
E-mail address: mhcho@snu.ac.kr (M. Cho).

http://dx.doi.org/10.1016/j.compstruct.2016.07.051
0263-8223/

layer-wise theories yield an inefficient analysis of laminates with several hundreds of layers, despite their great performance in the prediction of both displacement and stress.

To overcome the drawback of aforementioned theories, the zigzag theories have been developed by employing zigzag distribution through the thickness for the in-plane displacement fields. The refined first-order shear deformation theory (RFSDT) has been introduced by adding the piecewise linear variation function into the in-plane displacement field of FSDT [7,8]. To improve the accuracy, the efficient higher-order plate theory (EHOPT) [9,10], which meets the demands for accuracy and efficiency by combining the general cubic polynomial term and the linear zigzag distributions in the in-plane displacement form is introduced. Then, the free condition at top and bottom surfaces as well as the continuity condition at the interfaces between layers of transverse shear stresses is employed to reduce the number of unknown variables. Therefore, the EHOPT accurately predicts both global and local behaviors. Moreover, the number of unknowns is five, which is the same as that of FSDT, and does not depend upon the number of layers. The development of EHOPT for various applications such as finite element analysis, thermal-electro-mechanical problems, stability and dynamic analyses for both plate and shell models can be found in the serial papers [11–20].

To take the simplicity of FSDT and the accuracy of EHOPT, Kim and Cho [21,22] developed an enhanced first-order shear deformation theory (EFSDT) by assuming the transverse shear energy of both theories to be equivalent. Therefore, the finite element implementation of EFSDT is the same to that of FSDT. Besides, Dr. Hodges's group and Dr. Cho's group provided FSDT-like theories, that is, variational-asymptotic plate and shell theory (VAPAS) and EFSDT and showed their performances [23,24].

On the other hand, in the viscoelastic analysis, the Boltzmann superposition principle causes the constitutive equation to be expressed in the time-dependent integral form [25–32]. This time-dependent constitutive equation requires a great amount of computing time and resources if the behaviors of the viscoelastic composite structures are given by the numerical procedures based on the Taylor or trapezoidal time integration methods, whose computational accuracies depend on a time step Δt. Hence, to obtain the solution at the time t, enormous computational resources are required because time integration solutions must be made at each time step. In particular, the accuracy of the solution significantly decreases for long-term response problems. Therefore, the approaches that directly solve the time-dependent equations are not efficient for the prediction of long-term viscoelastic behaviors such as creep and relaxation processes. To overcome these limitations, some studies [33–36] have introduced Laplace or Fourier transforms from the time domain to the transformed domain. After calculating the unknown field variables in the Laplace domain, the responses in the real time domain are obtained by numerical inverse Laplace transformation [37–42]. Therefore, the efficiency of the computation is guaranteed by these methods for long-term response problems of viscoelastic analysis.

Previously, the analytical approach of EHOPT for viscoelastic composite laminates has been investigated only for simply sup-

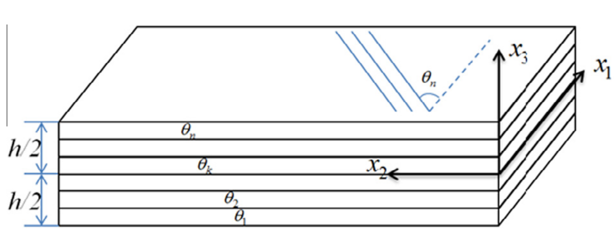

Fig. 1. Geometry and coordinates of the rectangular laminated plates.

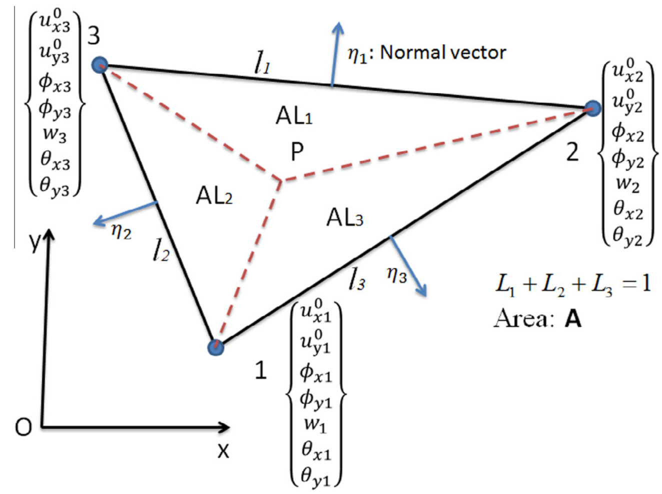

Fig. 2. Geometry and coordinates of a triangular element.

Table 1
Laminate stack sequences (SS: Simply Supported, CL: Clamped).

Laminate	Laminate orientation	Thickness h^k/h	Boundary condition	Applied loading
L1	[0/90/0]	[1/3;1/3;1/3]	SS	Sinusoidal
L2	[0/90/0/90/0]	[1/5;1/5;1/5;1/5;1/5]	SS	Sinusoidal
L3	[0/core/0]	[1/10;8/10;1/10]	SS	Sinusoidal
L4	[0/90/0]	[1/3;1/3;1/3]	CL	Uniform

Table 2a
Time-dependent function of the relaxation modulus of the GY 70/339 composite material.

p	b_p	τ_p
0	0.669825×10^{-1}	∞
1	0.813977×10^{-2}	$5.516602214 \times 10^{2}$
2	0.484272×10^{-1}	$1.494783951 \times 10^{4}$
3	0.710360×10^{-1}	$5.288067476 \times 10^{5}$
4	0.114155	$1.846670914 \times 10^{7}$
5	0.102892	$5.253922053 \times 10^{8}$
6	0.146757	$1.799163029 \times 10^{10}$
7	0.148508	$4.761315266 \times 10^{11}$
8	0.150514	$1.477467149 \times 10^{13}$
9	0.696426×10^{-1}	$4.976486103 \times 10^{14}$
10	0.729459×10^{-1}	$8.174141919 \times 10^{15}$

Table 2b
Time-dependent function of the relaxation modulus of the core.

p	b_p	τ_p
0	0.3844	∞
1	0.0806	0.9196
2	0.0172	0.9812e+01
3	0.0429	0.9527e+02
4	0.0491	0.9432e+03
5	0.0647	0.9207e+04
6	0.0753	0.89974e+05
7	0.0888	0.8685e+06
8	0.0874	0.8514e+07
9	0.1096	0.7740e+08