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Research Paper

Modelling the non-coaxiality of soils from the view of cross-anisotropy

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ABSTRACT

Material anisotropy is widely recognized as the fundamental reason that causes non-coaxiality between principal directions of stress and plastic strain increment. In order to model the non-coaxiality of soils from the view of cross-anisotropy, the anisotropic transformed stress method is introduced. By replacing the ordinary stress with the anisotropic transformed stress, and adopting a normal flow rule in the transformed stress space, non-coaxiality can be reflected simply within the framework of conventional elastoplastic constitutive theory. As an example, the unified hardening model is extended to be cross-anisotropic by this method, and then used to predict the non-coaxiality.

1. Introduction

In the conventional elastoplastic constitutive theory, principal directions of plastic strain increment are assumed to be coaxial with principal directions of stress. This assumption is called postulate of coaxiality, which facilitates the development of constitutive models.

However, the applicability of the postulate of coaxiality is challenged by an increasing number of tests on anisotropic soils. Early in 1970, Roscoe [1] indicated that during simple shear tests on Leighton Buzzard sand, principal axes of the strain increment did not coincide with those of stress before dilatancy. Using the directional shear cell, Wong and Arthur [2] found that if an inherently anisotropic sample was loaded with the major principal stress deviating from the direction of deposition, the major principal strain increment would deviate farther away from the deposition direction. In addition to simple shear apparatus and directional shear cell, hollow cylinder apparatus, which is capable of controlling both magnitudes and directions of principal stresses, is widely used to investigate soil behaviors under complex stress paths [3,4]. Tests with fixed principal stress axes [5,6] manifested coaxiality when the major principal stress was parallel or orthogonal to the deposition direction, i.e., if the angle between the major principal stress direction and the deposition direction, α_σ, was 0° or 90°, the angle between the direction of the major principal plastic strain increment and that of deposition, $\alpha_{d\varepsilon^p}$, was also 0° or 90°, respectively. When $0^\circ < \alpha_\sigma < 90^\circ$, however, $\alpha_\sigma \neq \alpha_{d\varepsilon^p}$ and non-coaxiality existed. The largest deviation angle between the principal stress direction and the principal strain increment direction was about 11° for dense Portaway sand and occurred when $\alpha_\sigma = 30^\circ$ [7]. Non-coaxiality was much more remarkable in principal stress rotation, with the non-coaxial angle $(\alpha_{d\varepsilon^p} - \alpha_\sigma)$ up to 30° for Toyoura sand [8]. The degree of non-coaxiality was influenced by many factors, such as the material anisotropy, density, loading history, stress ratio and intermediate principal stress [9–11].

There have been several attempts on developing constitutive models to describe the non-coaxial behaviors of soils. The 'double sliding, free rotating model' proposed by de Josselin de Jong [12] was among the first that could account for non-coaxiality. However, this model presumed that the material was perfectly plastic. Within the framework of elastoplastic constitutive theory, a vertex-like yield surface was adopted by Rudnicki and Rice [13] to describe strain localization into the shear band. Compared with the conventional model, this model obtained a plastic strain increment which was not only related to stress but also to stress increment, so that in some cases principal directions of stress and plastic strain increment would not be coaxial. Inspired by this work, Papamichos and Vardoulakis [14], Yang and Yu [15], Yu and Yuan [16] developed non-coaxial models in which the plastic strain increment was decomposed into a coaxial component and a non-coaxial component. The coaxial component was still determined by the current stress according to conventional models, while the non-coaxial component, as a correction term to readjust the plastic flow direction, depended on stress increment. An extra mechanism was introduced to calculate the non-coaxial plastic strain

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Nomenclature

b	intermediate principal stress coefficient
e	void ratio
e_0	initial void ratio
f	yield function
F_{ij}	fabric tensor
g	plastic potential function
H	hardening parameter
I_i	stress invariant of the ordinary stress tensor ($i = 1, 2, 3$)
M	critical/characteristic state stress ratio q/p
$\overline{M}$	critical/characteristic state stress ratio $\tilde{q}/\tilde{p}$ ($= \bar{q}_c/\bar{p}$)
M_f	failure stress ratio q/p
$\overline{M}_f$	failure stress ratio $\tilde{q}/\tilde{p}(= \bar{q}_c/\bar{p})$
p	mean stress of the ordinary stress tensor
$\bar{p}$	mean stress of the modified stress tensor
$\tilde{p}$	mean stress of the anisotropic transformed stress tensor
$\tilde{p}_0$	intercept of the initial yield surface on the $\tilde{p}$-axis
q	deviatoric stress of the ordinary stress tensor
$\bar{q}$	deviatoric stress of the modified stress tensor
$\bar{q}_c$	deviatoric stress at the triaxial compression state in the space of $\bar{\sigma}_{ij}$
$\tilde{q}$	deviatoric stress of the anisotropic transformed stress tensor
α_σ	angle between the major principal direction of σ_{ij} and the z-axis
$\alpha_{\bar{\sigma}}$	angle between the major principal direction of $\bar{\sigma}_{ij}$ and the z-axis
$\alpha_{\tilde{\sigma}}$	angle between the major principal direction of $\tilde{\sigma}_{ij}$ and the z-axis
$\alpha_{d\sigma}$	angle between the major principal direction of $d\sigma_{ij}$ and the z-axis
$\alpha_{d\varepsilon}$	angle between the major principal direction of $d\varepsilon_{ij}$ and the z-axis
$\alpha_{d\varepsilon^p}$	angle between the major principal direction of $d\varepsilon_{ij}^p$ and the z-axis
β	non-coaxial angle
γ_{oct}	octahedral shear strain
δ_{ij}	Kronecker delta
Δ	principal value of the fabric tensor
ε_{ij}	strain tensor
ε_{ij}^p	plastic strain tensor
ε_v^p	plastic volumetric strain
$\tilde{\eta}$	stress ratio $\tilde{q}/\tilde{p}$
θ	Lode's angle of the ordinary stress tensor
κ	slope of the swelling line
λ	slope of the normal compression line
Λ	plastic multiplier
ν	Poisson's ratio
σ_{ij}	ordinary stress tensor
$\bar{\sigma}_{ij}$	modified stress tensor
$\tilde{\sigma}_{ij}$	anisotropic transformed stress tensor

increment. This method was then extended to three-dimensional stress space by Qian et al. [17], and was integrated into the rotational hardening subloading surface model by Tsutsumi and Hashiguchi [18]. Besides, the description of non-coaxiality could also be realized by introducing a special mapping rule in which the projection center was relocatable (Li and Dafalias [19]), or the mapping direction was along the direction of stress increment (Gutierrez et al. [8], Lashkari and Latifi [20]). These models are flexible, and can get good agreement with test results. However, non-coaxial behaviors are not considered based on material anisotropy, which is widely recognized as the fundamental reason of non-coaxiality [2,3,5–11,19,20].

In this paper, non-coaxial behaviors are modeled by considering the cross-anisotropy of soils. A newly proposed method, called anisotropic transformed stress method [21], is introduced. Using this method, anisotropic constitutive models can be developed and can account for non-coaxiality automatically and simply, without introducing any extra mechanism.

2. Coaxiality in conventional elastoplastic constitutive theory

In the three-dimensional space (z, x, y), a complete description of the stress state of an element in the ground needs six independent stress components: σ_z, σ_x, σ_y, τ_{zx}, τ_{xy} and τ_{yz}. And correspondingly, six independent strain components, ε_z, ε_x, ε_y, γ_{zx}, γ_{xy} and γ_{yz}, are needed to fully describe the strain state. The work of a constitutive model is to provide a stiffness matrix to connect these stress components with strain components. However, it is very cumbersome to establish the model under such large dimension directly. For convenience, three stress invariants, σ_1, σ_2 and σ_3 (or p, q and θ, or I_1, I_2 and I_3), are used in the yield functions and plastic potential functions of conventional elastoplastic constitutive models. A small-scale stiffness matrix between principal stresses and principal strains can be obtained first. Then by rewriting the three invariants as functions of the six stress components, dimensionality is increased and the stiffness matrix connecting all the stress and strain components can be derived.

The extension from three invariants to six components, however, implies a limitation to the principal directions of stress and strain, because the dimensionality is not equivalent. This limitation is the very postulate of coaxiality. To illustrate this, let us take the modified Cam-clay model [22] as an example. Its plastic potential function is expressed as

$$g = q^2 + M^2p^2 - Cp = 0 \tag{1}$$

where mean stress $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$; deviatoric stress $q = \sqrt{(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2}/\sqrt{2}$; M = critical state stress ratio q/p; C controls the size of the plastic potential surface. And the flow rule is

$$d\varepsilon_i^p = \Lambda \frac{\partial g}{\partial \sigma_i} \tag{2}$$

where $d\varepsilon_i^p$ = principal value of the plastic strain increment ($i = 1, 2, 3$); Λ = plastic multiplier. In order to obtain the stiffness matrix under the general state, p and q are rewritten as $\sigma_{ii}/3$ and $\sqrt{3(\sigma_{ij} - p\delta_{ij})(\sigma_{ij} - p\delta_{ij})/2}$ (δ_{ij} = Kronecker delta), respectively. And the plastic strain increment $d\varepsilon_{ij}^p$ is calculated by extending the flow rule to be

$$d\varepsilon_{ij}^p = \Lambda \frac{\partial g}{\partial \sigma_{ij}} \tag{3}$$

However, the stress components σ_{ij} extended from p and q are not independent of each other. Six components have only two degrees of freedom which are equal to the degree of freedom of the stress invariants p and q. Because in the case of plane strain state ($\tau_{xy} = \tau_{xz} = 0$), the principal stress direction can be easily observed in the $(\sigma_z - \sigma_y)/2 \sim \tau_{zy}$ plane, p and q are expressed as functions of $(\sigma_z - \sigma_y)/2$, τ_{zy} and $(\sigma_z + \sigma_y)/2$ as