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Research Paper

Identification of representative slip surfaces for reliability analysis of soil slopes based on shear strength reduction

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ABSTRACT

Although a slope may have numerous potential slip surfaces, its failure probability is often governed by several representative slip surfaces (RSSs). Previous efforts mainly focus on the identification of circular RSSs based on limit equilibrium methods. In this paper, a method is suggested to identify RSSs of arbitrary shape based on the shear strength reduction method. Monte Carlo simulation is used to generate a large number potential slip surfaces. The RSSs are identified through analyzing the failure domains represented by these samples. A kriging-based response surface model is employed to enhance the computational efficiency. These examples shows that the RSSs may not always be circular, and that the suggested method can effectively locate the RSSs without making prior assumptions about the shape of the slip surfaces. For the examples investigated, the system failure probabilities computed based on the shear strength reduction method are comparable to, but not the same as those computed based on the limit equilibrium methods. The suggested method significantly extends our capability for identifying non-circular RSSs and hence probabilistic slope stability analysis involving non-circular slip surfaces.

1. Introduction

A slope may have many potential slip surfaces. As sliding along any slip surface causes failure, the failure probability of a slope is not the same as that along a single slip surface. In recent years, there is an increasing interest in the system reliability of slopes [6,7,15,20]. It is now recognized that although a slope may have many potential slip surfaces, its system failure probability is only governed by a small number of critical slip surfaces, which are called representative slip surfaces (RSSs) in the literature (e.g., [33,34]). After the RSSs are identified, the system reliability can be conveniently calculated based on these slip surfaces. In addition, the RSSs also provide useful insights into the major failure modes of the slope and the contribution from different slip surfaces to the system failure probability, which are very instrumental in slope failure hazard mitigation. How to identify RSSs is an important aspect of probabilistic slope stability analysis.

Previously, Ching et al. [6] pioneered the system reliability of slopes considering a large number of potential slip surfaces. Zhang et al. [33] suggested a method to search circular RSSs based on analyzing the reliability indexes of different slip surfaces and correlation coefficients of factors of safety (FOSs) among different slip surfaces. Ji and Low [20] developed a local search method for identifying RSSs. Cho [9] used a barrier method to search RSSs. Li et al. [22] showed how to identify the RSSs through analyzing the correlation coefficients among FOSs of different sip surfaces. In these methods, the potential slip surfaces are all assumed *circular*, and are most suitable when the rotational failure mechanism dominates. One exception might be the recent work by Zeng et al. [30], in which Spencer's method combined with the genetic algorithm is used to search RSSs of arbitrary shape. In implementing such an algorithm, it is important that the slip surfaces considered are exhaustive such that all important failure surfaces are taken into account. Currently, studies on RSS identification are mainly based on limit equilibrium methods considering circular slip surfaces.

Ching et al. [7] highlighted the challenges in identifying general RSSs based on the limit equilibrium analysis, and suggested that non-circular RSSs may be better found based on approaches like the shear strength reduction method (e.g., [11,18]). The shear

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strength reduction method is another widely used technique for slope stability analysis. Compared with limit equilibrium methods, the shear strength reduction method can identify critical slip surfaces without making prior assumptions about the shape of the slip surfaces (e.g., [4,26]), hence may be potentially an ideal tool for identifying non-circular RSSs. Indeed, the shear strength reduction method has been used for reliability analysis of soil slopes considering the spatial variability of soil properties (e.g., [17,19]). Nevertheless, little work has been done on how to locate RSSs based on the shear strength reduction method.

The objective of this paper is thus to suggest a method to identify RSSs of arbitrary shapes based on the shear strength reduction method for system reliability of soil slopes. This paper is organized as follows. First, the general idea of identifying RSSs based on the shear strength reduction method is introduced. Then, a kriging-based response surface method is suggested to make the suggested idea computationally feasible. Finally, the suggested method is illustrated and verified with three examples. The method suggested in this paper can serve as a practical tool for identifying non-circular RSSs and for system reliability analysis of slopes with arbitrary shapes.

2. Identification of representative slip surfaces

2.1. Generation of potential slip surfaces of arbitrary shapes

To identify the RSSs, the first step is to generate a large number of potential slip surfaces. For circular RSSs, the potential slip surfaces can be generated by varying the centers and radii of the slip surfaces. Such an idea is not feasible for non-circular slip surfaces, as it is difficult to parameterize a general slip surface with arbitrary shape. The shear strength reduction method, in which the shear strength parameter are gradually reduced until equilibrium cannot be maintained (e.g., [31,11,18]), is an effective approach for searching for the most critical slip surface without prior assumptions about its shape (e.g., [4,26]). Also, it does not require assumptions on the interslice shear force distribution [5]. Griffiths and Lane [18] suggested that the widespread use of the shear strength reduction method should be seriously considered over the limit equilibrium methods. Over the past decades, the shear strength reduction method has been implemented in several commonly used geotechnical software. One possible limitation of the shear strength reduction method is that it is computationally more demanding, particularly when a large number of repeated analyses is required. In this paper, the shear strength reduction method will be used to generate slip surfaces of arbitrary shapes. How to enhance the computational efficiency will be addressed in a later section.

Let $\boldsymbol{\theta}$ denote the uncertain shear strength parameters in a slope stability analysis. Given an arbitrary value of $\boldsymbol{\theta}$, a slip surface can be generated by substituting the value of $\boldsymbol{\theta}$ into the shear strength reduction model. If a large number samples of $\boldsymbol{\theta}$ is available, a large number of potential slip surfaces can then be generated by substituting each of these samples into the shear strength reduction model. In this study, the potential slip surfaces are generated using Monte Carlo simulation, i.e., samples of $\boldsymbol{\theta}$ are first generated according to its probability density function (PDF), and the potential slip surfaces are obtained from the shear strength reduction method using these samples as input. If the number of samples is large, the generated slip surfaces will be exhaustive. Generating potential slip surfaces in such a way bypasses the need to parameterize a general slip surface, which is very difficult. It should be noted that, although the shear strength reduction method is used here, other slope stability methods may also be used within the approach suggested in this paper, as long as such methods can search the most critical slip surface with an arbitrary shape in a reliable and robust way.

2.2. Interpretation of samples in the failure domain

Cho [9] showed that the identification of RSSs is equivalent to a multi-design point problem in structural reliability theory, as shown in Fig. 1a. The essence of slope reliability analysis based on first order reliability method through RSSs is to use a set of hyperplanes passing through the design points to approximate the actual limit state function. Therefore, the identification of RSSs is equivalent to locating multiple design points in the slope reliability problem. In the method suggested here, a large number of samples of $\boldsymbol{\theta}$ will be generated. If the number of samples is large, there will be a large number of points densely populated around the limit state function, as shown in Fig. 1b. The multi-design points may then be identified from these points. Since each sample represents a possible slip surface, the RSSs can then be identified after the design points are found. Therefore, the problem of identifying RSSs is then reduced to how to identify the design points among a large number of points in the failure region. As design points are all on the limit state surface, the following discussion will focus on failure points close to the limit state surface.

Consider an arbitrary point A on the limit state surface in the reduced space, as shown in Fig. 1c. Its distance to the origin O is indeed the reliability index characterizing the failure domain defined by the hyperplane passing through A, which can be calculated using the following equation (e.g., [1])

$$\beta_A = \sqrt{\mathbf{u}_A^T \mathbf{u}_A} \tag{1}$$

where $\mathbf{u}_A$ is the coordinates of A in the reduced space. Thus, a shorter distance denotes a smaller reliability index and hence a more probable failure point.

Consider another point B on the limit state surface with $\beta_B > \beta_A$, i.e., point B is less critical than the point A, as shown in Fig. 1c. If the two points are very close, the failure domain defined by the hyperplane passing through point B can be largely represented by that of point A, i.e., if two points are very close, the effect of point B on reliability evaluation can be represented by point A. Let α denote the angle between lines OA and OB. As can be seen from Fig. 1c, the value of α will be close to 0 if points A and B are close, and will increase as two points are more far away. Thus, α can be viewed as a measure of distance between points A and B in the reduced space. In reliability theory, α is related to the correlation coefficient between the hyperplanes passing through points A and B, ρ_{AB}, as follows (e.g., [1]):

$$\rho_{AB} = \cos\alpha = \frac{\mathbf{u}_A^T \mathbf{u}_B}{\beta_A \beta_B} \tag{2}$$

where $\mathbf{u}_B$ is the coordinate of B in the reduced space. As points A and B are closer, the value of α will be closer to zero, and in such a case the value of ρ_{AB} will also increase. Similarly, the value of ρ_{AB} will decrease as points A and B are more separated. Thus, ρ_{AB} is a function of the distance between two points on the limit state functions.

2.3. Identification of representative slip surfaces

Previously, Zhang et al. [33] suggested a method to identify circular RSSs based on analyzing the design points of a large number of circular slip surfaces found based on first order reliability analysis. The key idea for finding RSSs governing the system reliability analysis is as follows, i.e., if the correlation coefficient between design points of two circular RSSs is large, it indicates the failure of the two circular RSSs are highly correlated, and the failure domain of the less critical slip surface can be represented by that of the more critical one. As can be seen from the analysis described in the previous section, the failure domains defined by two points