Construction and Building Materials 148 (2017) 398–410

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Fracture energy-based model for average crack spacing of reinforced concrete considering size effect and concrete strength variation

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HIGHLIGHTS

- The crack spacing of RC members is analyzed using the fracture energy criterion.
- The influence of compressive strength on crack spacing is first explained.
- The size effect of the crack spacing is first examined for RC members.
- A simple formula for crack spacing is proposed with a reasonable level of accuracy.

ARTICLE INFO

Article history:
Received 4 November 2016
Received in revised form 23 April 2017
Accepted 6 May 2017
Available online 13 May 2017

Keywords:
Average crack spacing
Crack width
Size effect
Concrete strength variation
Reinforced concrete
Fracture energy
Finite element analysis

ABSTRACT

The average crack spacing is a key parameter for an accurate evaluation of the crack width of reinforced concrete (RC) members. According to the test results in the existing literature, both the variations of the concrete strength and the size effect are critical factors influencing the average crack spacing. However, available prediction models for the average crack spacing cannot give satisfactory results in simulating both factors. Based on the finite-element (FE) analysis and the fracture-energy criterion, a theoretical method considering the influence of concrete strength variation and size effect is first proposed. It is assumed that a micro-crack will grow into a visible crack if and only if the energy release exceeds the fracture energy of the effective cracking area. Therefore, the average crack spacing can be predicted by equating the energy release, which is obtained by the three-dimensional FE model of concrete subjected to bond stress, to the fracture energy of the effective cracking area. In addition, from the proposed model, the characteristic length of concrete is found to be the most important material parameter for average crack spacing of RC members. Subsequently, a database including 136 test specimens is established to sufficiently validate the proposed model. The influence of various key factors on the average crack spacing is discussed in detail. Finally, simplified prediction formulas for average crack spacing of RC members are proposed considering both concrete strength variation and size effect. Comparisons indicate that both the proposed theoretical model and the simplified formulas have sufficient accuracy.

1. Introduction

Cracking of concrete is one of the most important phenomena that indicate the durability of reinforced concrete (RC) structures. Cracking of RC members significantly influences the structural performance, including tensile and bending stiffness, energy absorption capacity, ductility, and corrosion resistance of reinforcement [1]. In a routine cracking analysis of RC members, the average crack spacing of RC members L_m should be determined in advance, and then the crack width of RC members could be calculated as recommended by existing literatures [1–6] as:

$$w_m = \varepsilon_{cr} \cdot L_m \tag{1}$$

$$\varepsilon_{cr} = \varepsilon - \varepsilon_e \tag{2}$$

where w_m denotes the average crack width; ε_{cr} denotes the cracking strain of concrete; ε denotes the total tensile strain of concrete; and ε_e denotes the elastic portion of tensile strain of concrete.

Presently, by using nonlinear FE analysis considering the tension-softening effect of concrete [7], the cracking strain of concrete can be predicted with satisfactory accuracy. However, existing prediction formulas for average crack spacing show

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http://dx.doi.org/10.1016/j.conbuildmat.2017.05.082
0950-0618/

Nomenclature

List of symbols

A	effective concrete cracking area of each rebar
A_e	effective cracking area of concrete
A_s	total area of rebar in tension
B	width of specimen
c	smallest cover thickness (excluding rebar radius)
d	distance from compression surface to the center of rebar for beams
d_a	maximum aggregate size
D	diameter of rebar
D_0	a constant with the dimension of length
f_t	tensile strength of concrete
f_c'	average compressive strength of 150 mm × 300 mm concrete cylinder specimen
G_F	fracture energy of concrete
H	height of specimen
l_{ch}	characteristic length of concrete
L_m	average crack spacing
L_{exp}	experimental average crack spacing
L_{pre}	predicted average crack spacing
L_{max}	maximum crack spacing
L_{min}	minimum crack spacing
N_0	total number of rebar in tension
N	number of rebar in bottom row in tension
S	minimum rebar distance (neglecting rebar radius)
U	strain energy of concrete in finite element model
V	geometric portion of strain energy of concrete $V = U \cdot E_c/\tau^2$
w_m	average crack width
α	bond stress divided by tensile strength of concrete
ε_{cr}	cracking strain of concrete
σ	standard deviation
τ	bond stress between rebar and concrete
ΔU	energy release by forming a new crack
ρ	reinforcement ratio of cross section
μ	mean value
ω	coefficient of variation

substantial deviations from experimental results, making it difficult to accurately evaluate the average crack width. For example, in the commercial software ATENA, the average crack spacing is recommended to be input by the user so that the crack width can be calculated precisely. Researchers have made substantial efforts to predict the average crack spacing, and most of these efforts are divided into three categories:

1.1. Strength criterion

This method assumes that the crack formation requires the maximum stress in concrete to be equal to the tensile strength. Watstein and Parsons [8] summarized the strength criterion and concluded that the average crack spacing was proportional to D/ρ (where D is the rebar diameter, and ρ is the reinforcement ratio). Michael and Kirstein [9] first introduced the concept of effective cracking area in the strength criterion. Piyasena et al. [10,11] combined the two-dimensional FE method with strength criterion to derive semi-empirical prediction formulas for the average crack spacing of flexural RC members. However, Base [12] reported that although the bond strength of modern deformed bars and plain bars differed by four times, the average crack spacing and crack width differed only by a relatively minor extent which violated the prediction by the strength criterion. Furthermore, Hognestad [2] concluded that the average crack spacing was strongly dependent on the rebar diameter D for plain bars and old-type deformed bars without longitudinal ribs. However, since the release of the ASTM Designation: A-305 [13] for rebar deformations, the modern American deformed bars which possess longitudinal ribs were used worldwide. Hognestad [2] found out that the average crack spacing was less dependent on D for modern deformed bars with longitudinal ribs, which also violated the results of the strength criterion.

1.2. Data regression method

Based on test results, Broms [14–16] first proposed that the average crack spacing mainly depended on t (cover thickness including rebar radius). Oh and Kang [1] used statistical regression analysis to investigate 129 data points of average crack spacing and 747 data points of crack width of beams and derived simplified formula. In addition, the formulas provided in most widely used design codes [4,5,17] also lied in this category. However, the data regression method is highly subjective depending on the researchers' skill and choice of input parameters, and the influence of the concrete strength and size effect on the average crack spacing in the previous experiments was generally neglected.

1.3. Fracture energy criterion

Bažant and Oh [18] analyzed the average crack spacing and width using both the energy criterion of fracture mechanics and the strength criterion. For energy criterion, crack formation was treated by considering that the entire crack formed simultaneously in one finite jump and the average crack spacing was halved. This method assumed that the bond stress caused the stress in concrete to be given by an inclined stress line with slope of 0.7. This model simplified the concrete section to a square region with a single bar at the center while maintaining the cross-sectional area as constant; afterwards, the simplified solution of the energy release due to cracking is calculated based on the assumed stress filed. According to Bažant and Oh's model [18], the aspect ratio of RC specimen has no influence on crack spacing. However, Lee and Kim's test [19] recently showed that while keeping the rebar area and the concrete area unchanged, the aspect ratio of tensile specimens strongly influences the average crack spacing, which contradicted with Bažant and Oh's [18] model.

In general, existing methods have not covered the significant influences of the following two critical factors: (i) *concrete compressive strength*. Lee and Kim [19] recently tested 35 rectangular direct tension specimens with cylinder concrete compressive strength ranging from 25 MPa to 80 MPa, and the test results showed that the enhancement of the concrete strength reduced the average crack spacing and width significantly. Another experimental evidence is as follows: Tammo and Thelandersson [20] recently used short direct tension specimens and kept the crack spacing constant by reserving cracks before loading. In their research, increasing concrete strength had no effect on the observed crack width, neither close to the bar nor at the concrete surface. On the other hand, it is widely recognized that when the crack spacing was not kept constant by preserving cracks, the enhancement of concrete strength will reduce the crack width