Journal of Natural Gas Science and Engineering 35 (2016) 1079–1087

Contents lists available at ScienceDirect

Journal of Natural Gas Science and Engineering

journal homepage: www.elsevier.com/locate/jngse

A novel approach to simulate the stress and displacement fields induced by hydraulic fractures under arbitrarily distributed inner pressure

Yu Peng, Yongming Li, Jinzhou Zhao *

State Key Laboratory of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu 610500, China

ARTICLE INFO

Article history:
Received 8 June 2016
Received in revised form
18 September 2016
Accepted 20 September 2016
Available online 21 September 2016

Keywords:
Hydraulic fracturing
Induced stress
Induced displacement
Pressure drop
Asymmetrical propagation
Fluctuating geo-stress

ABSTRACT

Stress and displacement fields induced by hydraulic fractures have been studied by a lot of researchers since they seriously affect the hydraulic fracture geometry. In previous studies, the distribution of inner pressure was assumed to be uniform or diminishing. Therefore, taking the complexly distributed inner pressures, such as geo-stresses with random fluctuations, into consideration in the classical models was unrealizable. A convenient and stable superposition model is developed to simulate the induced stress and displacement fields around artificial or natural fractures under arbitrarily distributed inner pressures in conjunction with complex variable method in theory of elasticity, which includes Westergaard Stress Function Method. The new model is validated by the analytical solution of induced displacement field around a fracture under linear load. It could be inferred that the superposition model is always convergent with a large enough number of discrete segments since the structure of the superposition model is similar to that of the numerical integrations. Some conclusions are drawn from the simulation results of the stress and displacement fields around fractures, when taking the inner pressure drop, asymmetrical propagation and fluctuating geo-stress into consideration. In consideration of the inner pressure drop in artificial fracture, narrower fracture width will form and the risk of early screen-out will increase significantly. Hence, in this case a greater pad volume or a higher pumping rate is needed. The symmetry of the induced stress and displacement fields would be broken by the asymmetrical propagation of artificial fracture. However, the difference of the results computed by classical model and superposition model is negligible in the far field (where $y > a$). The influence of fluctuating geo-stress on displacement distribution is non-ignorable since the most important displacement computing data is fracture width, which is the double of the displacement at $y = 0$.

1. Introduction

Hydraulic fracture propagation patterns, initiation pressures of adjacent fractures, interactions between artificial and natural fractures, when to refracture and the opportunities to form fracture networks in multi-cluster staged well fracturing are all strongly affected by the stress and displacement fields. A large number of literature described how they affect the hydraulic fracturing processes.

Atkinson and Eftaxiopoulos, (2002) have studied the initiation patterns of cement wells, when the steel/cement and the cement/rock interfaces under pure bond or pure slip condition respectively. Analytical and numerical results show that the induced stress of artificial fractures may have a severe influence on their own propagation patterns, like others observed in the simulations of finite or boundary element methods (Chen, 2012; Weng, 2015). Besides the interaction between different artificial fractures, there are a lot of studies on the interaction between artificial and natural fractures, too. Several criterions are established for analyzing the phenomenon of hydraulic fracture cross interfaces or discontinuities by Blanton (1986), Warpinski and Teufel, (1987) andRenshaw and Pollard (1995). The post-crossing propagation is simulated by the non-planar propagation model (Cheng et al., 2015) and 2D Displacement Discontinuity Method (DDM) (Xie et al., 2016).

In the optimization of hydraulic fracturing processes, Guo et al. (2015) have built up a 3D fracture propagation model based on

* Corresponding author.
E-mail address: zhaojz@swpu.edu.cn (J. Zhao).

http://dx.doi.org/10.1016/j.jngse.2016.09.054
1875-5100/

finite-element method to simulate multi-cluster staged well fracturing, and propose a cluster space optimization method which considers the fracture geometry of each cluster and the induced stress field between the fractures. Lu et al. (2015) have investigated the influence of pulsating hydro-fracturing on the stress disturbance in a coal seam by a stress-disturbance numerical model assumed in an infinite elastic formation. Zangeneh et al. (2015) have studied the impact of stress shadows on horizontal hydraulic fractures from adjacent lateral wells by a discontinuum-based distinct-element method, the results reveal that the interactions between artificial and natural fractures have the potential to change the size and effectiveness of the hydraulic fracturing stimulation by changing the induced stress field around the secondary well. Lin et al. (2016) have pointed out the existence of stress field interference in different stages of fracturing operations leads to the change in the magnitude and direction of stress field after each staged fracturing. Ren et al., (2016) simulation results of fracture network propagation have shown that the geological conditions and engineering treatments will determine what kind of fracture network is formed, due to the extreme variation of induced stress.

All the results above have one thing in common, they are dominated by the stress field induced by artificial and natural fractures. Therefore, calculating the induced stress and displacement accurately and rapidly is of great importance for the simulation of fracture initiation, propagation and how to formation a fracture network.

In fact, induced stress field of a primary fracture is systematically studied in refracturing. In last decade, with the prosperous development of refracturing design and execution, researchers have paid more and more attention on the induced stress fields modeling. A lot of factors such as injection (or production) rate, pore pressure variation, temperature stress and seepage field have been coupled into the calculation model by finite or boundary element method (Zhai and Sharma, 2007; Aghighi and Rahman, 2010a, 2010b; Altmann et al., 2010; Roussel, 2011). However there are still some defects in the current models. Due to the flow resistance in the fracture and the differences of geo-stresses between different layers, the distribution of net pressure in the fracture is complex and fluctuating, which is realized in the simulation of fracture propagation long time ago (Clifton and Abou-Sayed, 1981; Palmer and Carroll, 1983; Warpinski and Teufel, (1987);Morales and Abou-Sayed, 1989). Although the stress and displacement fields around artificial fractures under diminishingly distributed inner pressure could be simulated by some models and simulators (Taghichian et al., 2014; Weng, 2015; Kumar and Ghassemi, 2015) with complicated program and long duration of simulation, most of the physical models of induced stress and displacement calculation are still under uniform inner pressures similar to classical works (England and Green, 1963; Warpinski and Branagan, 1989).

In conclusion, it is widely accepted that the distributions of geo-stresses are not uniform or changing continuously, there are always some fluctuations exist. The assumption of uniformly or diminishingly distributed inner pressure will restrain the practicality of the previous models. Besides that, a convenient and stable calculation model of induced stress and displacement, which could take complexly distributed inner pressure into consideration, is still absent. In order to solve the above problems, a computation first and separation last approach (computing before separating) is employed to apply the results of complex variable method in theory of elasticity. Then, a semi-analytical superposition model is established for simulating the induced stress and displacement around artificial or natural fractures under arbitrarily distributed inner pressure. In this model, every reasonable inner pressure could be taken into consideration, whether or not it is continuous or piecewise, regardless of its integrability. It was verified that the new model is accurate and the classical model built by Westergaard and Paris (Tada et al., 2000) is only a special case of the superposition model. As field cases, the influence of inner pressure drop, asymmetric propagation and fluctuating geo-stress on stress and displacement distributions are simulated by the semi-analytical superposition model.

1.1. Complex variable method in theory of elasticity

Complex variable method was established by some famous mathematicians and physicists such as Muskhelishvili (1977), Paris and Sih (1964) and Tada et al. (2000). Among all the achievements relate to fracture mechanics, Westergaard Stress Function Method (Sadd, 2005) is the most representative work. Westergaard suggested that the stress function could be expressed by the single and double integral of an analytic function Z:

$$\varphi = \mathrm{Re}\overline{\overline{Z}} + y\mathrm{Im}\overline{Z} \tag{1}$$

where,

$$\overline{Z} = \int Z dz$$

$$\overline{\overline{Z}} = \int \overline{Z} dz$$

where φ is the stress function of mode-I cracks, Z is an analytic function satisfies the boundary conditions, z is the complex variable of Z.

It is proved that φ is a bi-harmonic function in conjunction with Cauchy-Riemann Eq. (2) (Rudin, 1987). Therefore, φ is an appropriate stress function. The detailed proof is in appendix.

$$\begin{cases} \dfrac{\partial \mathrm{Re}Z}{\partial x} = \dfrac{\partial \mathrm{Im}Z}{\partial y} \\ \dfrac{\partial \mathrm{Re}Z}{\partial y} = -\dfrac{\partial \mathrm{Im}Z}{\partial x} \end{cases} \tag{2}$$

According to the characteristics of stress function, the stress field could be expressed as (under plane strain condition):

$$\begin{cases} \sigma_x = \mathrm{Re}Z - y\mathrm{Im}Z' \\ \sigma_y = \mathrm{Re}Z + y\mathrm{Im}Z' \\ \sigma_z = \mu(\sigma_x + \sigma_y) = 2\mathrm{Re}Z \\ \tau_{xy} = -y\mathrm{Re}Z' \end{cases} \tag{3}$$

where σ_x, σ_y, σ_z are the normal stress components, τ_{xy} is the shear stress, μ is Poisson's ratio.

By substituting Eq. (3) into constitutive equation and geometric equation, the displacement field under plane strain condition is obtained:

$$\begin{cases} u_x = \dfrac{1+\mu}{E}\left[(1-2\mu)\mathrm{Re}\overline{Z} - y\mathrm{Im}Z\right] \\ u_y = \dfrac{1+\mu}{E}\left[2(1-\mu)\mathrm{Im}\overline{Z} - y\mathrm{Re}Z\right] \end{cases} \tag{4}$$

where u_x, u_y are the displacement components, E is the elastic modulus.

1.2. Basic unit model

Many of the stress and displacement models are based on the simplest physical model of crack in which there is only one pair of