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Backscatter in stratified turbulence

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ABSTRACT

In this paper, kinetic and potential energy transfers around a spectral test filter scale in direct numerical simulations of decaying stratified turbulence are studied in both physical and spectral domains. It is shown that while the domain-averaged effective subgrid scale energy transfer in physical space is a net downscale cascade, it is actually a combination of large values of downscale and upscale transfer, i.e. forward- and backscatter, in which the forward scatter is slightly dominant. Our results suggest that spectral backscatter in stratified turbulence depends on the buoyancy Reynolds number Re_b and the filtering scale Δ_{test}. When the test filter scale Δ_{test} is around the dissipation scale L_d, transfer spectra show spectral backscatter from sub-filter to intermediate scales, as reported elsewhere. However, we find that this spectral backscatter is due to viscous effects at vertical scales around the test filter. It is also shown that there is a non-local energy transfer from scales larger than the buoyancy scale L_b to small scales. The effective turbulent Prandtl number spectra demonstrate that the assumption $Pr_t \approx 1$ is reasonable for the local energy transfer.

1. Introduction

Large eddy simulations (LES) is an approach for decreasing the computational costs of direct numerical simulations (DNS) of turbulent flows. In LES, the large energy-containing eddies are directly resolved but subgrid scale (SGS) eddies are parametrized. Most SGS parameterizations such as the Smagorinsky [1] and Kraichnan [2] models are based on the Richardson [3] energy-cascade hypothesis, which argued that turbulent kinetic energy is generated at large scales and dissipated at small scales. Richardson's prediction may be valid for the average kinetic energy cascade, but might not be accurate in the local sense. Locally, the energy cascade is the net outcome of forward scatter, i.e. the energy transfer from large to small scales, and the backscatter, which is the reverse energy transfer from small to large scales. For example, Piomelli et al. [4] and Domaradzki et al. [5] have shown that forward- and backscatter are of the same order of magnitude in turbulent channel flow and isotropic decaying turbulence, respectively.

The dynamic SGS scheme, proposed by Germano et al. [6], has been designed to improve the performance of purely dissipative eddy viscosity SGS schemes such as the Smagorinsky [1] model by considering a time- and space-dependent dynamic Smagorinsky coefficient c_s with negative values corresponding to backscatter. However, averaging c_s over a homogeneous direction is often required to avoid numerical instabilities (e.g. [7–9]). Information about the dynamics of the local energy transfer is therefore lost, and in practice, the averaging procedure removes the local effect of backscatter, because the averaged c_s is usually positive (e.g. [7,9–12]).

Stratified turbulence is a model for turbulence in the atmospheric mesoscale and oceanic sub-mesoscale, at which fluid motions are strongly affected by stratification but weakly affected by the Earth's rotation (e.g. [13]). The presence of stratification leads to the generation of anisotropic features such as pancake vortices, which in turn lead to the development of different length scales and spectral slopes in the horizontal and vertical directions (e.g. [14–19]). Recently, the dynamics of energy transfer between large and sub-filter scales in stratified turbulence has been studied in wavenumber space [20,21]. Using DNS of decaying stratified turbulence, Khani and Waite [20] studied the dynamics of horizontal and vertical energy transfer around the Ozmidov scale. It was shown that stratification leads to a non-local energy transfer from large to small horizontal scales when an anisotropic horizontal test cutoff k_c is employed [20]. In addition, the spectral eddy viscosity based on the vertical kinetic energy shows negative values when the flow is subjected to very stable stratification, leading to negative effective turbulent Prandtl number [21].

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The performance of different eddy viscosity SGS models (the Smagorinsky, dynamic Smagorinsky and Kraichnan models) has recently been studied in LES of stratified turbulence with isotropic grid resolution [10,22]. In all cases, it is shown that LES must resolve the buoyancy scale $L_b = 2\pi u_{rms}/N$ to capture the fundamental large scale characteristics of stratified turbulence. Here, u_{rms} and N are the root-mean-square velocity and buoyancy frequency. The resolution criterion for LES of stratified turbulence depends on the SGS model: it requires $\Delta < 0.47L_b$ for the Kraichnan model, $\Delta < 0.24L_b$ for the dynamic Smagorinsky model and $\Delta < 0.17L_b$ for the regular Smagorinsky model (where Δ is the grid spacing, [10,22]). These criteria are obtained by studying the capability of different SGS models to capture three fundamental features of stratified turbulence: layered structures that break down into Kelvin–Helmholtz (KH) instabilities and small-scale turbulence when shear is large; horizontal wavenumber energy spectra with an approximately $-5/3$ power law at large scales along with a bump (or shallowness) around k_b; and the resolution of regions with small and negative Richardson number, which demonstrate the presence of KH instabilities, overturning and small-scale turbulence. These fundamental features have been reported in several DNS and hyperviscosity simulations of stratified turbulence (e.g. [14,15,17–20,23–26]). The importance of L_b implies that LES of stratified turbulence does not require resolution of the smaller Ozmidov scale $L_o = 2\pi(\epsilon/N^3)^{1/2}$, where ϵ is the kinetic energy molecular dissipation rate [10,22]. As a result, the potential of employing LES with much coarser grids than DNS is promising.

In physical space, backscatter can be calculated from DNS data by filtering velocity fields, and directly measuring the sub-filter scale (SGS) momentum tensor τ_{ij}. Following Piomelli et al. [4], backscatter may be defined by negative values of the effective SGS dissipation rates ϵ_{SGS} and ε_{SGS}, written as

$$\epsilon_{SGS} = -2\tau_{ij}\bar{S}_{ij}, \qquad \varepsilon_{SGS} = -h_j\frac{\partial\bar{\rho}}{\partial x_j}, \tag{1}$$

where $\bar{S}_{ij}$ is the filtered rate of strain. A similar procedure was used recently to analyse SGS backscatter in DNS of reacting turbulence [27], which found that backscatter depends on the dynamics of reacting flows and hence is not just a random and intermittent process. The dynamics of backscatter has not been studied for stratified turbulence. Indeed, the physical mechanisms underlying the dynamics of energy transfer are not completely understood in this context.

In this paper, the dynamics of forward and inverse energy transfer around the Ozmidov scale in DNS of stratified turbulence is studied at different buoyancy Reynolds numbers and using different test filters. In addition, the spectral kinetic and potential energy transfer, and the effective turbulent Prandtl number, are analysed. The governing equations of motion and formulations for analysing DNS diagnoses are presented in Section 2. Section 3 presents the methodology used, and Section 4 presents the results and discussion. Conclusions are given in Section 5.

2. Governing equations

The non-dimensional Boussinesq equations are

$$\frac{\partial\boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p - \frac{1}{{Fr_\ell}^2}\rho\boldsymbol{e}_z + \frac{1}{Re_\ell}\nabla^2\boldsymbol{u}, \tag{2}$$

$$\nabla\cdot\boldsymbol{u} = 0, \tag{3}$$

$$\frac{\partial\rho}{\partial t} + \boldsymbol{u}\cdot\nabla\rho - w = \frac{1}{Re_\ell Pr}\nabla^2\rho, \tag{4}$$

where $\boldsymbol{u} = (u, v, w)$ is the velocity vector, ρ and p are the density and pressure perturbations, respectively; Re_ℓ, Fr_ℓ, and Pr are the initial Reynolds, Froude and Prandtl numbers, respectively, which are defined in terms of the initial velocity and length scales. The density perturbation is nondimensionalized in terms of the constant background density gradient and the initial length scale. We can define the test-filtered variables by applying a filtering operator to the DNS results, e.g. for velocity

$$\bar{\boldsymbol{u}}(\boldsymbol{x}, t) = \int_D G(\acute{\boldsymbol{x}}, \boldsymbol{x})\boldsymbol{u}(\acute{\boldsymbol{x}}, t)\mathrm{d}\acute{\boldsymbol{x}}, \tag{5}$$

where G is the filtering function and D is the spatial domain. Hence, the governing equations (2)–(4) can be rewritten for the test-filtered variables as follows:

$$\frac{\partial\bar{u}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{u}_i\bar{u}_j) = -\frac{\partial\bar{p}}{\partial x_j} - \frac{1}{{Fr_\ell}^2}\bar{\rho}\boldsymbol{e}_z + \frac{1}{Re_\ell}\frac{\partial^2\bar{u}_i}{\partial x_j\partial x_j} - \frac{\partial\tau_{ij}}{\partial x_j}, \tag{6}$$

$$\frac{\partial\bar{u}_i}{\partial x_i} = 0, \tag{7}$$

$$\frac{\partial\bar{\rho}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{\rho}\bar{u}_j) - \bar{w} = \frac{1}{Re_\ell Pr}\frac{\partial\bar{\rho}}{\partial x_j\partial x_j} - \frac{\partial h_j}{\partial x_j}, \tag{8}$$

where

$$\tau_{ij} = \overline{u_iu_j} - \bar{u}_i\bar{u}_j, \qquad h_j = \overline{\rho u_j} - \bar{\rho}\bar{u}_j, \tag{9}$$

are the SGS momentum and density fluxes which are known since DNS resolves both the test-filtered and sub-test-filter scales. Using the measured SGS momentum flux τ_{ij}, we can calculate the SGS dissipation field ϵ_{SGS} through Eq. (1), which gives the local rate of energy transfer between the test-filtered scales and the sub-test-filter motions. Following Piomelli et al. [4], if ϵ_{SGS} is positive, then the kinetic energy transfers from the test-resolved to the SGS motions (forward scatter); however, if ϵ_{SGS} is negative, kinetic energy is transferred in the opposite direction (backscatter). The SGS dissipation rate can be decomposed into forward- and back-scatter contributions as $\epsilon_{SGS} = \epsilon^+ + \epsilon^-$, where

$$\epsilon^\pm = \frac{1}{2}(\epsilon_{SGS} \pm |\epsilon_{SGS}|). \tag{10}$$

Similarly, forward- and backscatter of potential energy is

$$\varepsilon^\pm = \frac{1}{2}(\varepsilon_{SGS} \pm |\varepsilon_{SGS}|). \tag{11}$$

In the wavenumber domain, assuming periodic boundary conditions and a sharp spectral filter with wavenumber k_c, we can use the measured SGS fluxes in (9) to calculate the kinetic and potential effective SGS energy transfer following Pope [9] as

$$T_k(k|k_c, t) = \frac{1}{2}\langle F_j\hat{\bar{u}}_j^* + F_j^*\hat{\bar{u}}_j\rangle_k, \tag{12}$$

$$T_p(k|k_c, t) = \frac{1}{2}\langle J\hat{\bar{\rho}}^* + J^*\hat{\bar{\rho}}\rangle_k, \tag{13}$$

where F_j and J are the Fourier coefficients of the SGS fluxes $\partial\tau_{ij}/\partial x_j$ and $\partial h_j/\partial x_j$, respectively, where $*$ is complex conjugate and the angle bracket $\langle\cdots\rangle_k$ denotes summing of the Fourier modes over spherical shells of constant radius k. Here, $k = |\boldsymbol{k}|$ and $\boldsymbol{k} = (k_x, k_y, k_z)$ is the three dimensional wavevector. The $|k_c$ notation in (12)–(13) underlines that the transfers are computed relative to a particular filter wavenumber [9]. The physical and spectral SGS energy transfer are related: the sum of the SGS transfer spectra equals the domain-averaged SGS dissipation rate, written as

$$\sum_k T_k(k|k_c, t) = \epsilon_{SGS}(k_c, t), \tag{14}$$

$$\sum_k T_p(k|k_c, t) = \varepsilon_{SGS}(k_c, t), \tag{15}$$