European Journal of Mechanics B/Fluids 59 (2016) 90–98

Contents lists available at ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

Particle-laden viscous channel flows: Model regularization and parameter study

Lennon Ó. Náraigh [a,b,*], Ricardo Barros [c]

[a] School of Mathematical Sciences, University College Dublin, Belfield, Dublin 4, Ireland
[b] Complex and Adaptive Systems Laboratory, University College Dublin, Belfield, Dublin 4, Ireland
[c] Mathematics Applications Consortium for Science and Industry (MACSI), Department of Mathematics and Statistics, University of Limerick, Limerick, Ireland

ARTICLE INFO

Article history:
Received 14 January 2016
Received in revised form
19 May 2016
Accepted 19 May 2016
Available online 26 May 2016

Keywords:
Particle-laden viscous flow
Inclined channel
Diffusive-flux model
Regularized model
Counter-current flow
Flow reversal

ABSTRACT

We characterize the flow of a viscous suspension in an inclined channel where the flow is maintained in a steady state under the competing influences of gravity and an applied pressure drop. The basic model relies on a diffusive-flux formalism. Such models are common in the literature, yet many of them possess an unphysical singularity at the channel centreline where the shear rate vanishes. We therefore present a regularization of the basic diffusive-flux model that removes this singularity. This introduces an explicit (physical) dependence on the particle size into the model equations. This approach enables us to carry out a detailed parameter study showing in particular the opposing effects of the pressure drop and gravity. Conditions for counter-current flow and complete flow reversal are obtained from numerical solutions of the model equations. These are supplemented by an analytic lower bound on the ratio of the gravitational force to the applied pressure drop necessary to bring about complete flow reversal.

1. Introduction

Particles suspended in viscous flow occur in a wide variety of applications. In certain technical operations (for example drilling oil wells), it is important to be able to predict the properties of the suspension as a function of flow rate, particle size, etc., with a view to controlling the operation in real time [1]. There is therefore a strong motivation to develop accurate models to characterize the hydrodynamics of the suspension. In this work we introduce a simple model to characterize the flow of suspension in an inclined channel under equilibrium conditions. The modelling framework is the diffusive-flux model. Such models certainly abound in the literature—and many of them exhibit an unphysical singularity in the shear rate at the channel centreline. The main goal of this work therefore is to introduce a self-consistent regularization that removes this singularity. A second goal is to carry out a detailed parameter study based on the regularized model to fully characterize the hydrodynamics as a function of the dimensionless parameters in the problem—both in the horizontal and inclined cases. Before doing this, we place our work in the context of the existing literature on the subject.

There are at least two distinct approaches to modelling a suspension of dense particles in a (Newtonian) liquid. In the first approach, called the suspension balance model, the averaged dynamics of the suspended particles are described in a statistical-mechanics formalism; this description is then coupled to the fluid mechanics of the problem. This model was first proposed in Ref. [2]. A review of the model (along with various refinements thereto) can be found in Refs. [3,4]. The model involves mass and momentum equations for the particle phase (averaged over a test volume) and the mixture (again averaged over a test volume), leading to four evolutionary equations in the first instance. Both momentum equations involve particle-phase and mixture stress tensors respectively, and the particle momentum equation further involves a hydrodynamic drag force, meaning that three constitutive relations are required for closure. The closure is achieved by modelling the hydrodynamic drag force and various viscous terms. The particle-phase shear stress term is modelled by the introduction of an auxiliary variable (the particle-phase 'temperature'), leading to a set of five coupled evolution equations. A simpler approach that makes predictions of comparable accuracy to the suspension-balance model is the diffusive-flux model, first introduced in Ref. [5] but based partly on earlier work [6] (see

* Corresponding author.
E-mail address: onaraigh@maths.ucd.ie (L.Ó. Náraigh).

http://dx.doi.org/10.1016/j.euromechflu.2016.05.005

also Ref. [7]). The idea here is to focus entirely on the mixture for the hydrodynamic model, together with an advection–diffusion equation for the volume fraction ϕ of the particles. The particle flux in this equation is then modelled according to the collision dynamics of the particles, to include shear-induced migration, viscous migration, and gravitational settling.

In the present work, the diffusive-flux model is adopted. The reasons for this choice are manifold: the diffusive flux framework is both conceptually and analytically straightforward, and involves only a handful of parameters, all of which can be estimated from benchmark cases. Although it has shortcomings (e.g. a lack of a complete description of the anisotropy of particle interactions, leading to incorrect predictions in complex geometries [4]), it produces good agreement between theory and experiments for flow profiles and volume profiles in relatively simple geometries, e.g. horizontal pressure-driven pipe/channel flows, as well as in rotating shear flow [5]. The present work is focused on one such simple geometry, namely channel flow. Finally, it has been shown that the suspension balance and diffusive-flux models share the same basic framework, the main difference being the choice of closure relations for the different parameters [8].

In spite of the tractability of the diffusive-flux model, in its basic form it cannot be applied to fully-developed flow in an inclined channel. This is because the model develops a singularity wherever the shear rate vanishes. A review of the literature shows that this problem is overcome in certain highly specific contexts—e.g. resorting to a symmetry and placing the singularity at the centreline of a horizontal pipe/channel [5], or exploiting the specific properties of interfacial flows and placing the singularity at a free surface [9]. Yet the geometry of an inclined channel prevents these solutions from being applied in the present context. Furthermore, existing efforts to overcome these issues are incomplete. Ref. [7] looks at inclined flows, but only in the context of Brownian particle diffusion, which is not relevant at the high Péclet numbers with which this work is concerned and in any case is not a diffusive-flux model. Ref. [10] introduces a regularization of the full diffusive-flux model that removes the singularity through the introduction of a collision rate proportional to a shear rate that is averaged over a particle radius. However, the averaging is accomplished using effectively an L^1 norm, which on mathematical grounds is not optimal, as such an approach does not completely regularize the model. (In detail, the shear stress is regularized by an averaging technique involving an L^1 norm, yet the L^1 norm is not differentiable, hence the lack of a complete removal of the singularity by this means.) More importantly, the regularized model is not applied to inclined flows. Therefore, a main aim of the present work is to derive a regularization procedure that fully heals the singularity inherent in diffusive-flux models. This then enables a full parameter study for inclined flows that takes account of the different flow regimes that arise as a result of the competing effects of the pressure drop and gravity, as well as the bulk volume fraction and the channel inclination.

This work is organized as follows. In Section 2 the standard diffusive-flux model from the literature is summarized. A diffusive-flux model specific to steady-state operations in inclined channel flow is presented in Section 3, along with a regularization to heal the singularity that would otherwise occur where the shear-rate vanishes. Results based on this approach are presented in Section 4, including a detailed parameter study outlining the conditions under which different flow regimes are observed. The observed flow regimes comprise regular flow (both fluid and particles flowing upwards), countercurrent flow, and complete flow reversal. We discuss the application of our model to suspending fluids with non-Newtonian rheology in Section 5, wherein concluding remarks are also given.

2. General theoretical framework

In this section we summarize the full diffusive-flux theoretical framework existent in the literature, with a view later on to subject this model to a regularization technique to enable a full parameter study of the steady-state flow in an inclined channel. The starting point is a momentum equation for the velocity $\boldsymbol{u}(\boldsymbol{x}, t)$ of a parcel comprising a mixture of particles and suspending fluid:

$$\rho(\phi)\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\right) = \nabla\cdot\boldsymbol{T} + \rho(\phi)\boldsymbol{g} \tag{1}$$

where ϕ is the particle-phase volume fraction, $\boldsymbol{g}$ is the acceleration due to gravity and $\boldsymbol{T}$ is the mixture stress tensor. We start by assuming that the total mixture stress tensor is given by $\boldsymbol{T} = -p\boldsymbol{I} + \boldsymbol{\sigma}$, in which p is the pressure, and $\boldsymbol{\sigma} = \mu(\phi)\dot{\boldsymbol{\gamma}}$. Here $\dot{\boldsymbol{\gamma}}$ is the rate-of-strain tensor with components $\dot{\gamma}_{ij}$, where

$$\dot{\gamma}_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}. \tag{2}$$

The corresponding unsigned local rate of strain is given by

$$\dot{\gamma} = \sqrt{\dot{\gamma}_{ij}\dot{\gamma}_{ij}}, \tag{3}$$

where we sum over repeated indices. Furthermore, the density is given by

$$\rho(\phi) = \rho_p\phi + \rho_f(1-\phi), \tag{4}$$

where ρ_f is the constant fluid density and ρ_p is the particle density, also constant. This is supplemented by the incompressibility condition $\nabla\cdot\boldsymbol{u} = 0$.

The evolution of the volume fraction ϕ is given by a flux-conservative equation,

$$\frac{\partial\phi}{\partial t} + \boldsymbol{u}\cdot\nabla\phi = -\nabla\cdot\boldsymbol{J}_\phi, \tag{5}$$

where the particle flux $\boldsymbol{J}_\phi$ is modelled according to the collision dynamics of the particles, to include shear-induced migration, viscous migration, and gravitational settling. A classification of the collective particle dynamics provide a means of constituting the flux $\boldsymbol{J}_\phi$. The main effect to consider is shear-induced migration, which is based on the observation that in a dense suspension, particles that are transported by a shear flow will collide. The collision rate is proportional to $\phi\dot{\gamma}$, where $\dot{\gamma}$ is the (unsigned) local shear rate. Particles will move from regions where the collision rate is high to a nearby region where the collision rate is lower, meaning that there is a shear-induced contribution $\boldsymbol{J}_c$ to the total flux, with $\boldsymbol{J}_c \propto -\nabla(\phi\dot{\gamma})$. Ref. [5] gives the shear-induced flux as

$$\boldsymbol{J}_c = -D_c\phi a^2\nabla(\phi\dot{\gamma}), \tag{6}$$

where D_c is a dimensionless constant and a is the particle radius (a monodisperse suspension of identical spherical particles is assumed). A second effect is present in viscous flows, whereby particles will move into regions of lower viscosity, from regions of higher viscosity. In Ref. [5] this is modelled in such a way that the viscous contribution to the total flux is proportional to the ratio between the viscosity gradient (giving the direction of migration) and the local viscosity, giving a total contribution

$$\boldsymbol{J}_v = -D_v a^2\phi^2\dot{\gamma}\left(\frac{\nabla\mu}{\mu}\right), \tag{7}$$

where D_v is another dimensionless constant (both D_c and D_v can be thought of as $O(1)$ rate constants related to the collision frequency, and can be determined from experimental data, as in Ref. [5]).