European Journal of Mechanics B/Fluids 34 (2012) 91–96

Contents lists available at SciVerse ScienceDirect

European Journal of Mechanics B/Fluids

journal homepage: www.elsevier.com/locate/ejmflu

An exact solution of AC electro-kinetic-driven flow in a circular micro-channel

Ali Jabari Moghadam *

Department of Mechanical Engineering, Shahrood University of Technology, Shahrood, Iran

ARTICLE INFO

Article history:
Received 5 July 2011
Received in revised form
2 January 2012
Accepted 6 March 2012
Available online 17 March 2012

Keywords:
Micro-fluidics
AC electro-kinetic-driven flow
Navier–Stokes equations
Circular micro-channel
Analytical solution
Frequency

ABSTRACT

The electro-osmotic fully-developed flow in a circular micro-channel is studied under an alternating electrical field. An analytical approach based on the linearized Poisson–Boltzmann equation is selected to get an exact solution of the electrical potential inside the channel. An exact solution of the velocity distribution is then obtained by using the Green's function approach. The application of the electrical body force results in a rapid acceleration of the fluid within the double layer. If the diffusion time scale is much greater than the oscillation period (high frequency), the fluid within the double layer oscillates rapidly, while the bulk fluid remains almost stationary.

1. Introduction

In recent years, considerable progress has been made in the field of miniaturization. It is now effectively possible to miniaturize all kinds of systems—e.g., mechanical fluidic, electromechanical, or thermal-down to sub-micrometric sizes. In particular, the micro-fluidic systems have been developed in which a fluid circulates inside a miniaturized channel, named micro-channel, by applying an electrical field along it. The rapid expansion of the micro-fluidics field seems to be driven in part by the possibility of integration. The domain of integrated analysis systems has been designated as micro-total analysis systems, or also lab-on-a-chip systems. Generally, a lab-on-a-chip device has a network of micro-channels, electrodes, sensors and electrical circuits. The advantages of these labs on a chip include dramatically reduced sample size, much shorter reaction and analysis time, high throughput, automation and portability [1]. The electro-osmotic flow is usually preferred over the pressure-driven flow, because pumping a liquid through a very small channel requires applying very large pressure difference depending on the flow rate. Additionally, it does not require any external pump, but needs electrodes to control the flow field.

Among the researchers that worked on these phenomena, Anderson [2] studied the particle movement produced by non-uniform zeta potential in an electric field. The effect of inhomogeneously charged surfaces on electro-osmosis was reported by Ajdari [3]. Wang and Chen [4] investigated electro-osmosis in homogeneously charged micro- and nanoscale random porous media using mesoscopic simulation methods which involve a random generation-growth method for reproducing three-dimensional random micro-structures of porous media and a lattice Poisson–Boltzmann algorithm for solving the strongly nonlinear governing equations. Wang et al. [5] modeled physicochemical transport due to electro-osmosis of dilute electrolyte solutions through micro-porous media with granular random microstructures by a three-step numerical framework. They investigated the effects of porosity, ionic concentration, pH, and temperature on the electro-osmotic permeability through the granular micro-porous media.

Among the researchers that worked on DC electro-osmotic flows, Dutta and Beskok [6] presented analytical results for velocity distribution, mass flow rate, pressure gradient, wall shear stress, and vorticity in mixed electro-osmotic/pressure driven flows for two-dimensional straight channel geometry. Arulanandam and Li [7] studied the liquid movement in a rectangular micro-channel by electro-osmotic pumping. Soong and Wang [8] studied flow and heat transfer between two parallel plates.

AC electro-osmotic flows have been studied by some researchers. Among them, Kang et al. [9] solved the electro-osmotic flow problem in a cylindrical channel for only sinusoidal waveform by the Green's function method. Wang and Kang [10] presented a numerical solution based on coupled lattice Boltzmann methods for electro-kinetic flows in micro-channels. They also presented an analytical flow field model, based on a surface slip condition approach, for an axially applied AC electrical field in an infinitely wide micro-channel. Comprehensive models for such a slit channel have also been presented by Dutta and Beskok [11] who developed

* Fax: +98 273 3395440.
E-mail addresses: jm.ali.project@gmail.com, jabariali@rocketmail.com.

0997-7546/$ – see front matter
doi:10.1016/j.euromechflu.2012.03.006

an analytical model for an applied sinusoidal electric field, and Soderman and Jonsson [12] who examined the transient flow field caused by a series of different pulse designs. Erickson and Li [13] presented a combined theoretical and numerical approach to investigate the time periodic electro-osmotic flow in a rectangular micro-channel.

As an alternative to traditional DC electro-osmosis, a series of novel techniques have been developed to generate bulk flow using AC fields. For example, Green et al. [14] experimentally observed peak flow velocities on the order of hundreds of micrometers per second near a set of parallel electrodes subject to two AC fields, 180° out-of-phase with each other. The effect was subsequently modeled using a linear double layer analysis by Gonzalez et al. [15]. Using a similar principle, both Brown et al. [16] and Studer et al. [17] presented micro-fluidic devices that incorporated arrays of non-uniformly sized embedded electrodes which, when subject to an AC field, were able to generate a bulk fluid motion.

In this research, an exact solution of flow induced by unsteady applied electric fields inside a circular micro-channel has been developed. The closed-form solution of the momentum equation presented within the Debye–Huckel approximation can be used to get the velocity profiles due to applying any time-periodic electric fields. This kind of micro-channel with its particular applied electric field has its unique features and applications. Lab-on-a-chip devices having networks of micro-channels are miniaturized bio-medical or chemistry laboratories on a small glass or plastic chip. Applying electrical fields along micro-channels controls the liquid flow and other operations in the chip. These labs on a chip can duplicate the specialized functions as their room-sized counterparts, such as clinical diagnostics, DNA scanning and electro-phoretic separation.

2. Problem formulation

Consider a fully-developed flow inside a circular micro-channel that is produced by an electric field in the absence of any pressure gradients.

First of all, we must know the local net charge density per unit volume ρ_e at any point in the solution. This requires solving the EDL field [18]:

$$\nabla^2\psi = \frac{2\mathbb{Z}en_0}{\varepsilon}\sinh\left(\frac{\mathbb{Z}e\psi}{k_BT}\right) \tag{1}$$

where, ψ is the electrical potential.

For pure electro-osmotic fully-developed flows of incompressible fluids in circular micro-channels, the Navier–Stokes equations take the following form [19]:

$$\rho\frac{\partial V_z}{\partial t} = \mu\left(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r}\frac{\partial V_z}{\partial r}\right) - \rho_e E(\omega t) \tag{2}$$

where, V_z is the only non-zero velocity component along the channel, ρ and μ are the density and the viscosity of liquid, respectively, and $E(\omega t)$ is a general time-periodic function with a frequency $\omega = 2\pi f$ that describes the applied electric field strength.

Eqs. (1) and (2) are the governing equations of this problem. The boundary conditions are:

$$\begin{cases} r = 0: \dfrac{d\psi}{dr} = 0 \\ r = \Re: \psi = \zeta \end{cases} \tag{3}$$

$$\begin{cases} r = 0: \dfrac{\partial V_z}{\partial r} = 0 \\ r = \Re: V_z = 0 \end{cases} \tag{4}$$

where, $\Re$ and ζ are the channel radius and the zeta potential, respectively.

Consider the following dimensionless variables:

$$\begin{aligned} &R = \frac{r}{\Re}, \qquad \Psi = \frac{\mathbb{Z}e}{k_BT}\psi, \qquad \theta = \frac{\mu}{\rho\Re^2}t, \\ &\Omega = \frac{\rho\Re^2}{\mu}\omega, \qquad V = \frac{\mathbb{Z}e\mu}{\varepsilon E_z k_BT}V_z \end{aligned} \tag{5}$$

in which, E_z is a constant equivalent to the strength of the applied electric field. Introducing the above dimensionless variables into Eqs. (1) and (2) gives the following non-dimensional forms of the governing equations:

$$\nabla^2\Psi = (\kappa\Re)^2\sinh\Psi \tag{6}$$

$$\frac{\partial V}{\partial\theta} = \frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R} + (\kappa\Re)^2\sinh\Psi\, F(\Omega\theta) \tag{7}$$

where, $F(\Omega\theta)$ is a general periodic function of unit magnitude such that $E(\Omega\theta) = E_zF(\Omega\theta)$. κ is the Debye–Huckel parameter defined as follows:

$$\kappa = \left(\frac{2\mathbb{Z}^2e^2n_\infty}{\varepsilon\varepsilon_0k_BT}\right)^{1/2}. \tag{8}$$

The boundary conditions (3) and (4) also take the following dimensionless form:

$$\begin{cases} R = 0: \dfrac{d\Psi}{dR} = 0 \\ R = 1: \Psi = Z \end{cases} \tag{9}$$

$$\begin{cases} R = 0: \dfrac{\partial V}{\partial R} = 0 \\ R = 1: V = 0. \end{cases} \tag{10}$$

Eq. (6), under the condition that the double layer potential Ψ is small, can be linearized by the so-called Debye–Huckel approximation, yielding:

$$\frac{d^2\Psi}{dR^2} + \frac{1}{R}\frac{d\Psi}{dR} = K^2\Psi \tag{11}$$

in which, the constant K has been introduced to denote $\kappa\Re$ (electro-kinetic radius). The solution of (11) subject to the boundary conditions (9) is:

$$\Psi(R) = \frac{Z}{I_0(K)}I_0(KR) \tag{12}$$

where, $I_\nu(x)$ is the modified Bessel function of the first kind and order ν, satisfying the following modified Bessel function:

$$x^2y'' + xy' - \left(x^2 + \upsilon^2\right)y = 0. \tag{13}$$

In order to solve Eq. (7), the Debye–Huckel approximation is implemented to result in the following form of the equation:

$$\frac{\partial V}{\partial\theta} = \frac{\partial^2 V}{\partial R^2} + \frac{1}{R}\frac{\partial V}{\partial R} + \underbrace{K^2\Psi(R)\,F(\Omega\theta)}_{Q(R,\theta)}. \tag{14}$$

A Green's function approach is now used to find an analytical solution for the non-dimensional form of the motion Eq. (14). The Green's function and the boundary conditions become [20]:

$$\frac{\partial g}{\partial\theta} - \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial g}{\partial R}\right) = \frac{\delta(R-\ell)\,\delta(\theta-\tau)}{2\pi R} \tag{15}$$

$$\left\{\begin{matrix} \lim\limits_{R\to 0}|g(R,\theta;\ell,\tau)| < \infty \\ g(1,\theta;\ell,\tau) = 0 \end{matrix}\right\}, \quad 0 < R,\ \ell < 1,\ 0 < \theta, \tau \tag{16}$$