Experimental Thermal and Fluid Science 75 (2016) 220–227

Contents lists available at ScienceDirect

Experimental Thermal and Fluid Science

journal homepage: www.elsevier.com/locate/etfs

Dimensional analysis and experimental study of gas penetration depth model for submerged side-blown equipment

Ji Ma, Ping Zhou *, Wei Cheng, Yanpo Song, Pengyu Shi

School of Energy Science & Engineering, Central South University, Changsha 410083, China
Hunan Key Laboratory of Energy Conservation in Process Industry, Changsha 410083, China

ARTICLE INFO

Article history:
Received 3 June 2015
Received in revised form 29 January 2016
Accepted 29 January 2016
Available online 10 February 2016

Keywords:
Dimensional analysis
Submerged side-blown equipment
Water model experiment
Gas penetration depth

ABSTRACT

A dimensionless penetration depth equation related to modified Froude number, Reynolds number and density ratio of gas–liquid was established by dimensional analysis and Buckingham *pi* theorem. Based on the experimental data of side-blown penetration, extracted by MATLAB digital image processing algorithm in this scaled water model experiment, a dimensionless expression which serves as a convenient way to determine the penetration depth under different operation conditions was developed by the Least Square Estimation (LES) method. Meanwhile, detailed comparison of different empirical equations from this study and other related researches has also been sufficiently investigated. At last, the influence of each dimensionless number on the penetration depth was discussed by partial correlation analysis.

0. Introduction

The submerged side-blown method is extensively applied in the smelting of nonferrous metals [1,2] (e.g., lead, zinc, antimony and copper smelting), due to its characteristics of good raw material adaptability, large smelting capacity, low energy consumption and simple equipment construction. The flowing features of gas-melt in submerged side-blown furnaces play an important role in its technical and economic indicators e.g., productivity, energy consumption and cost [3]. Especially, the width of a furnace is dominated the gas penetration depth [4], i.e., the greater the gas penetration depth is, the wider the furnace size is generally designed to be, which is helpful to improve the productivity.

A clear progress in gas-melt feature studies can be observed over the last 35 years. Compared to the corresponding publications, studies concerning the gas behaviors in bath smelting (e.g., the formation of bubbles [5–7], the distribution of bubble size [8–11], gas holdup [12–14], behaviors of jet flow [15–21]) dominate the main research. Whereas features related to gas penetration behavior in side-blown furnace is still expected to be more sufficiently investigated.

Considering related literatures in further detail, the main factors influencing the gas penetration depth are inertial force of gas, nozzle diameter, density of liquid, gravitational acceleration and viscosities of gas and liquid. And several empirical equations for predicting the side-blown penetration depth were proposed by scaled experiment [17,20,22–30]. Iguchi et al. [27] and Chang and Sohn [29] presented different formulations for the relationship between dimensionless penetration depth and modified Froude number *Fr*′. Wang et al. [28] recommended a function of dimensionless penetration depth with modified Froude number *Fr*′ and liquid viscosity. But the dimensionless groups represented in his formulation were still expected to be more universal. Therefore, a comprehensive consideration of the whole factors should be analyzed to develop a novel empirical equation in predicting the penetration depth.

1. Dimensionless model of penetration depth

Generally, the penetration depth influenced by nozzle diameter, gravitational acceleration, viscosities of gas and liquid, gas inertial force, density of liquid was expressed as:

$$H = f(D, g, \upsilon_g, I_g, \upsilon_l, \rho_l)$$

In these kinds of studies, liquid is the continuous phase while the bubbles can be treated as discrete phase. In other words, more attentions should be focused on how the bubble motion is influenced by liquid, and the flow information of bubble interior is always negligible. Besides, gas viscosity has little contribution to the penetration depth, while the viscous force from liquid seems more significant. Thus, the viscosity of gas would be neglected,

---

[*] Corresponding author at: School of Energy Science & Engineering, Central South University, Changsha 410083, China.

http://dx.doi.org/10.1016/j.expthermflusci.2016.01.017
0894-1777/

Nomenclature

a	undetermined coefficients (–)
b	undetermined coefficients (–)
c	undetermined coefficients (–)
H	penetration depth (m)
h	liquid level (m)
Q	flow rate ($m^3\ h^{-1}$)
D	nozzle diameter (m)
g	gravitational acceleration ($m\ s^{-2}$)
u	velocity ($m\ s^{-1}$)
I	inertial force ($kg\ m\ s^{-2}$)
Fr'	modified Froude number (–)
Re	Reynolds number (–)
T	temperature (°C)
StD	standard deviation (–)
MAPE	mean absolute percentage error
Cal.	pertains to the calculated value
Exp.	pertains to the experimental data

Greek symbols

υ	kinematic viscosity ($m^2\ s^{-1}$)
η	dynamic viscosity ($N\ s\ m^{-2}$)
ρ	density ($kg\ m^{-3}$)
g	pertains to the gas phase
l	pertains to the liquid phase

and other factors were retained in this problem. Finally, the main influencing factors were written as Eq. (1), and summarized in Table 1.

$$H = f_1(D, g, \rho_l, I_g, \upsilon_l) \tag{1}$$

For Eq. (1), there were six physical variables, and they were expressed in terms of three independent physical units (M, L, T). The Buckingham *Pi* Theorem states that Eq. (1) can be rescaled into an equivalent dimensionless relationship having only 3 dimensionless parameters. In this study, the I_g, ρ_l, D were selected as independent physical units, therefore the dimensionless parameters could be written as:

$$\pi_1 = HI_g^{a_1}\rho_l^{b_1}D^{c_1} \tag{2}$$

$$\pi_2 = gI_g^{a_2}\rho_l^{b_2}D^{c_2} \tag{3}$$

$$\pi_3 = \upsilon_l I_g^{a_3}\rho_l^{b_3}D^{c_3} \tag{4}$$

Thus, Eq. (1) was rescaled into:

$$HI_g^{a_1}\rho_l^{b_1}D^{c_1} = f_2(gI_g^{a_1}\rho_l^{b_2}D^{c_2}, \upsilon_l I_g^{a_1}\rho_l^{b_3}D^{c_3}) \tag{5}$$

In terms of Eq. (2), substituting the physical variables with the basic dimension units would lead to:

$$\pi_1 = L \cdot (I_g LT^{-2})^{a_1} \cdot (I_g L^{-3})^{b_1} \cdot L^{c_1} \tag{6}$$

Since π_1 was a combination dimensionless group, thus the exponents a_1, b_1, c_1 must satisfy the following simultaneous equations:

$$\left.\begin{array}{l} a_1 + b_1 = 0 \\ a_1 - 3b_1 + c_1 + 1 = 0 \\ -2a_1 = 0 \end{array}\right\} \tag{7}$$

Thus a_1, b_1, c_1 were solved:

$$a_1 = 0, b_1 = 0, c_1 = -1$$

Therefore,

$$\pi_1 = HD^{-1} = H/D \tag{8}$$

Table 1
Dimension of gas penetration depth and its influencing factors.

No.	Parameter	Symbol	Unit	Dimension
1	Maximum penetration depth of gas	H	m	L
2	Nozzle diameter	D	m	L
3	Gravitational acceleration	g	$m\ s^{-2}$	LT^{-2}
4	Liquid density	ρ_l	$kg\ m^{-3}$	ML^{-3}
5	Inertial force of gas	I_g	$kg\ m\ s^{-2}$	MLT^{-2}
6	Liquid kinematic viscosity	υ_l	$m^2\ s^{-1}$	L^2T^{-1}

Similarly, π_2, π_3 were obtained respectively.

$$\pi_2 = gI_g^{-1}\rho_l D^3 \tag{9}$$

$$\pi_3 = \upsilon_l I_g^{-0.5}\rho_l^{0.5} \tag{10}$$

where $I_g = \rho_g \frac{\pi}{4} D^2 |u_g - u_l|^2$, u_g is the velocity of gas blowing and u_l is the velocity of liquid around nozzle. Then Eq. (9) would become as:

$$\pi_2 = \frac{4}{\pi} \frac{\rho_l g D}{\rho_g |u_g - u_l|^2} = \frac{4}{\pi} \frac{1}{Fr'} \tag{12}$$

where $Fr' = \rho_g |u_g - u_l|^2 / \rho_l g D$. In the same way, π_3 was obtained:

$$\pi_3 = \frac{4 \cdot \upsilon_l \cdot \rho_l^{0.5}}{\pi \cdot \left(\rho_g \cdot D^2 \cdot |u_g - u_l|^2\right)^{0.5}} = \frac{\pi}{4} \cdot \sqrt{\frac{\rho_l}{\rho_g}} \cdot \frac{1}{Re_l} \tag{13}$$

where $Re_l = |u_g - u_l| \cdot D/\upsilon_l$.

Substituting Eqs. (8), (12), (13) into Eq. (5):

$$H/D = f_3\left(\frac{4}{\pi}\frac{1}{Fr'}, \sqrt{\frac{\rho_l}{\rho_g}} \cdot \frac{1}{Re_l}\right) \tag{14}$$

Based on a general form of dimensionless group equation, Eq. (15) could be rewritten as:

$$H/D = c \cdot \left(\frac{1}{Fr'}\right)^a \cdot \left(\sqrt{\frac{\rho_l}{\rho_g}} \frac{1}{Re_l}\right)^b \tag{15}$$

Eq. (15) indicated that gas penetration depth of side-blown process depended on the modified Froude number, the combination of density ratio of two phase and Reynolds number. In order to find out the relation between dimensionless penetration depth and its influencing dimensionless factors, experiments should be designed and performed to determine the exponents of above equation.

2. Experiment

2.1. Experimental apparatus

A 1:8 bench-scaled tank model of typical smelting furnace with submerged side-blown system was developed for the experiment. The tank was constructed by 8 mm thick clear and transparent Plexiglas and water was utilized to model the molten metal. The equipment used in this investigation consisted of compressed gas cylinder, water tank and high-speed camera etc. Detailed arrangement was shown in Fig. 1.

Compressed N_2 cylinder was connected to the water tank through a decompression valve. A manometer and mass flowmeter were utilized to measure the pressure and mass flow rate of N_2,