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Slip flow forced convection through microducts of arbitrary cross-section: Heat and momentum analogy☆

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ABSTRACT

This work presents a theoretical approach, based on momentum and energy or Reynolds analogy, to investigate forced convection in microducts of arbitrary cross-section. **H1** boundary condition is assumed for gas flow in the slip-flow regime with further complication of a temperature jump condition assumption. It is shown that applying an analogy concept, one can relate the slip-flow results to those of no-slip/no-jump ones available in the literature. Present results for slip flow in microchannels of parallel plate, circular, triangular, trapezoidal, polygonal, rhombic, and rectangular cross-sections are found to be in close agreement with those in the literature. A further modification, based on Chilton-Colburn analogy is applied to enable the Prandtl number variation effects when the Prandtl number is not equal to unity.

1. Introduction

Microscale heat and fluid flow is of great importance not only for playing a key part in the biological systems, but also for its application in cooling electronic equipment, see [1–3]. Such small devices, however, show different behaviors compared to their macroscale counterparts in being associated with the inclusion of slip velocity and temperature jump, as noted by Tunc and Bayazitoglu [4], and also Sparrow and Haji-Sheikh's [5] pioneering work on velocity slip. One notes that gaseous flows at such small passage cannot be accurately predicted using classical continuum physics since such flow is associated with a nonzero fluid velocity at the solid walls where a difference between the gas-wall temperature prevails when $0.001 < Kn < 0.1$, e.g., slip flow behavior for which the Navier–Stokes and thermal energy equations should be combined with the slip flow condition and wall temperature jump so that the results can match experimental measurements, see [6–10].

This paper uses the analogy between momentum and thermal energy equation to relate the heat transfer to pressure drop for such slip flow. Furthermore, these slip flow results can be related to no-slip flow data for which theoretical results are available in the literature for a number of boundary conditions. Here, the **H1** case, in the terminology of Shah and London [11], which permits heat transfer-pressure drop analogy, is of interest. The **H1** boundary condition assumes a constant (independent of x) longitudinal heat flux while in each cross-section the wall temperature is constant independent of transverse and spanwise directions. Forced convection through microducts of arbitrary cross-sections is considered. According to Morini [12], four technologies exist to build microchannels each of which lead to different geometries for a micro-flow device. For instance, chemical etching directly on the silicon wafer, leaves cross-sectional shape to depend on a number of factors including the crystallographic nature of the silicon.

As such, this paper proposes a shorthand way of calculating slip flow forced convection from the already existing no-slip solutions. Application of this methodology to microchannels of parallel plate, circular, elliptical, rhombic, and rectangular cross-sectional shape is verified by comparing the results with those available in the literature. Furthermore, novel results pertaining to microchannels of hexagonal cross-sectional shape are reported following the application of the proposed heat and momentum transfer analogy. Effects of Prandtl number variation on heat transfer for microducts of semicircular and triangular cross-section are also investigated.

2. Analysis

Consider gas flow in a straight microconduit of arbitrary but axially-uniform cross-section as shown in Fig. 1. It is assumed that steady fully

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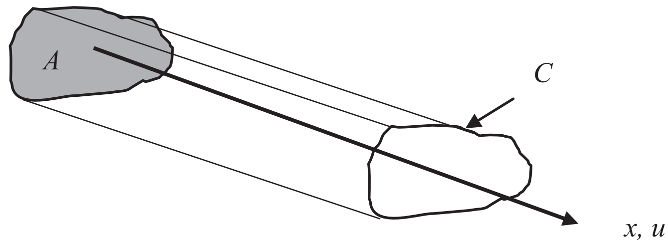

Fig. 1. Schematic view of the microduct with arbitrary cross-sectional shape. Flow direction (along x), microduct contour (C), and cross-sectional area A are shown.

developed unidirectional flow of a rarefied gas in the longitudinal (x) direction. The gas properties are assumed to be constant. Slip flow and temperature jump at the wall are also assumed to hold.

The slip velocity can be found as

$$u_s - u_w = \frac{F-2}{F} \lambda \frac{\partial u}{\partial n}|_{wall} \tag{1}$$

where u_s is the slip velocity, F is the tangential momentum accommodation coefficient, n denotes the coordinate which is normal to the wall, and λ is the molecular mean free path. On the other hand, the fluid temperature at the wall, T_s, can be in a different form that of the wall, T_w, i.e.,

$$T_s - T_w = \frac{F_t-2}{F_t} \frac{\lambda}{\Pr} \frac{2\gamma}{1+\gamma} \frac{\partial T}{\partial n}|_{wall}. \tag{2}$$

Here, F_t is the thermal accommodation coefficient, Pr is the Prandtl number, and γ is the specific heat ratio. The fully developed momentum equation, to be solved subject to the slip flow condition, Eq. (1), is given by

$$\frac{1}{U} \frac{dp}{dx} = \mu \nabla^2 u. \tag{3}$$

This can be integrated to give

$$\frac{C}{\mu} \frac{1}{U} \frac{dp}{dx} = \nabla u \tag{4}$$

where μ is the gas viscosity, the average velocity is U, C is the duct periphery as depicted in Fig. 1, and u is the dimensionless velocity.

Assuming constant fluid properties, the fully developed thermal energy equation reads

$$\frac{\partial(\rho c_p u T)}{\partial x} = k \nabla^2 T \tag{5}$$

where, following the application of the first law of thermodynamics to an element, the longitudinal temperature gradient takes this form

$$\frac{dT}{dx} = \frac{4q''}{\rho c_p U D_H}, \tag{6}$$

with $D_H = 4A/C$ being the hydraulic diameter and q'' the wall heat flux where ρ, k and c_p are the gas density, thermal conductivity and specific heat at constant pressure, respectively.

Now the thermal energy equation can be rearranged to read

$$\frac{4q'' u}{D_H U} = k \nabla^2 T. \tag{7}$$

Integrating over the fluid volume ($V = CD_H L/4$), with L being the duct length, one has

$$\frac{4q'' C}{D_H} = k \nabla T. \tag{8}$$

Using the dimensionless temperature and with

$$q'' = h(T_w - T_b) \tag{9}$$

one has

$$\frac{4Ch}{kD_H} = \nabla \theta \tag{10}$$

wherein $\theta = (T - T_w)/(T_b - T_w)$ is the dimensionless temperature.

Note the analogy between Eqs. (10) and (4). As such, one concludes

$$\frac{4Ch}{kD_H} = \frac{C}{\mu} \frac{1}{U} \frac{dp}{dx} \tag{11}$$

which, following the application of the Fanning friction factor f, one has

$$\frac{dp}{dx} = \frac{2f\rho U^2}{D_H} \tag{12}$$

leading to

$$\frac{Nu}{Po} = \frac{1}{2} \tag{13}$$

with Po, being the Poiseuille number defined as $Po = fRe$. Note that in all dimensionless numbers, the hydraulic diameter was selected as the length scale. One notes that Eq. (13) is pertinent to a case where Nu and Po are obtained from velocity and temperature fields which are affected by slip velocity and temperature jump.

The above equation allows for prediction of heat transfer based on pressure drop for slip flow through a micro-duct of arbitrary but uniform cross-section. For instance, results of Hooman [13] or those of Morini et al. [6] for the friction factor can be used to give Nusselt number without the need to solve the extra (thermal energy) equation. Alternatively, one can use the well-known Reynolds analogy for no-slip flow

$$\frac{Nu}{f\mathrm{Re}} = \frac{1}{2} \tag{14}$$

as a base case and relate the slip flow data to those of no-slip as

$$\frac{Nu}{Nu_0} = \frac{Po}{Po_0}. \tag{15}$$

Note that, one can either use the friction factor for slip flow as given by Morini et al. [6] to get the Nusselt number or get the Nu/fRe (goodness factor) values given by Shah and London [11] along with the fRe values in the literature, say those of Tamayol and Hooman [14] for ducts of arbitrary cross-section.

3. Results and discussion

Application of Eq. (15) above to different duct geometries will constitute a database for heat transfer of micro-ducts with arbitrary cross-sectional area. In an interesting study, Renksizbulut et al. [7] have reported numerical predictions for slip-flow forced convection through ducts of trapezoidal cross-section with rectangular ducts as limiting cases. More interestingly, their results are presented in the form of correlations which are cross-validated against those in the literature, e.g.,