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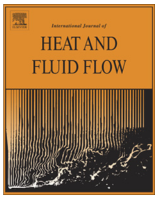

An intermittency model for predicting roughness induced transition

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ABSTRACT

An intermittency transport equation for RANS modeling, formulated in local variables, is extended for roughness-induced transition. To predict roughness effects in the fully turbulent boundary layer, published boundary conditions for k and ω are used. They depend on the equivalent sand-grain roughness height, and account for the effective displacement of wall distance origin. Similarly in our approach, wall distance in the transition model for smooth surfaces is modified by an effective origin, which depends on equivalent sand-grain roughness. Flat plate test cases are computed to show that the proposed model is able to predict transition onset in agreement with a data correlation of transition location versus roughness height, Reynolds number, and inlet turbulence intensity. Experimental data for turbine cascades are compared to the predicted results to validate the proposed model.

1. Introduction

1.1. The need for predicting rough wall, transitional boundary layers

It is well known that surface roughness can trip a boundary layer. Nevertheless, there are few data correlations or prediction methods for roughness induced transition. They are needed for many applications. For instance, to increase the efficiency of turbomachinery performance, designers must account for effects of surface roughness on both heat transfer and aerodynamic loss. Experimental data show that roughness on in-service turbine blades can cause a considerable increase in heat load. Arts et al. (1990) point out that, at moderate Reynolds numbers, a smooth vane can have transition occurring far downstream of the leading edge on the suction side, even with high inflow turbulence intensities; but, as the roughness height increases, the onset of transition gradually moves upstream, to the leading edge. When the boundary layer becomes turbulent, heat transfer can increase by a factor of 10 (Stripf et al., 2009a).

Boyle and Stripf (2009) mention that surface roughness generally decreases aerodynamic efficiency of a turbine blade cascade. But Boyle and Senyitko (2003) show that at low Reynolds numbers roughness improves aerodynamic efficiency, while at high Reynolds numbers roughness doubles vane loss. Therefore, to improve the efficiency at both low and high Reynolds numbers, it is necessary to properly represent the effects of roughness on the boundary layer.

1.2. Approaches to calculate a fully turbulent boundary layer on a rough wall

The model represents transition from laminar flow to turbulent flow over a rough wall. This requires a turbulence model that is applicable to a rough surface. In the present approach, the rough surface is replaced by an effective, smooth surface, on which new boundary conditions are imposed. They are a function of roughness height. One common approach to parameterize roughness is the equivalent sand grain roughness, which is adopted here.

Based on the sand grain roughness, Durbin et al. (2001) proposed a rough wall modification for the two layer $k-\epsilon$ model. An effective *displacement of the wall distance origin* was introduced and related to the sand grain roughness height through a calibration procedure. The effective displacement was related to a hydrodynamic roughness length, that is used to modify turbulence length scales and the boundary condition for ϵ. The following equation was used to blend between the smooth and fully rough boundary conditions for k:

$$k_w = \frac{u_*^2}{\sqrt{C_\mu}} \min\left[1, (r^+/90)^2\right], \tag{1.1}$$

where u_*, defined as $u_*^2 = (\nu + \nu_T)\partial_n U|_w$ for rough walls, is the friction velocity, and $C_\mu = 0.09$. r^+ is the dimensionless sand roughness height, ru_*/ν, where r is the dimensional roughness height.

Similar roughness boundary conditions for fully turbulent boundary layer have been proposed to extend the standard $k-\omega$ model: an early example is the Wilcox roughness modification (Wilcox, 1998); more recent models by Seo (2004) and Knopp

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et al. (2009) invoke the *displacement of origin* approach. While the Wilcox model requires a very fine mesh resolution and is not accurate for transitionally rough walls, the newer models give satisfactory results with near wall grid spacing similar to that for smooth walls.

Under fully rough conditions, the log-layer solution $k = u_*^2/\sqrt{C_\mu}$ extends to the effective wall origin, where the log-layer eddy viscosity $\nu_T = u_*\kappa(y + d_0)$ reduces to $u_*\kappa d_0$. Here d_0 is the effective displacement of the wall origin. d_0 can be determined analytically under fully rough conditions based on the shift of the velocity profile in the log-layer. This shift has been measured experimentally and fitted such that the new velocity profile can be written

$$U/u_* = 1/\kappa \log(y/r) + 8.5,$$

where $\kappa = 0.41$. Then, if d_0 is defined in terms of U by

$$U/u_* = 1/\kappa \log((y + d_0)/d_0),$$

the last two equations give

$$\frac{y + d_0}{d_0} = \frac{y}{r} e^{8.5\kappa}.$$

Using the approximation $d_0 \ll y$,

$$d_0 = e^{-8.5\kappa} r \approx 0.03r. \tag{1.2}$$

From the definition $\nu_T = k/\omega$, the boundary condition for ω under fully rough condition should be

$$\omega = \frac{u_*}{\sqrt{C_\mu}\kappa d_0}. \tag{1.3}$$

Generally, the ω boundary condition represented as

$$\omega_w = \frac{60\nu}{\beta y_{eff}^2}, \tag{1.4}$$

where $y_{eff} = \max[y_1, y_r]$ in Knopp's model and $y_{eff} = (y_1 + y_r)$ in Seo's. Here y_1 is the grid point next to the wall and

$$y_r = \frac{\nu}{u_*}\left(\frac{60\kappa\sqrt{C_\mu}}{\beta} d_0^+\right)^{1/2},$$

to agree with (1.3), where $\beta = 0.075$.

The variable d_0^+ is a function of r_+ that is obtained by calibration against the log-layer displacement, $\Delta U(r^+)$ (Durbin, 2009). Knopp proposes

$$d_0^+ = 0.03r^+ \times \min\left[1, \left(\frac{r^+}{30}\right)^{2/3}\right] \min\left[1, \left(\frac{r^+}{45}\right)^{1/4}\right] \min\left[1, \left(\frac{r^+}{60}\right)^{1/4}\right]. \tag{1.5}$$

Seo gives

$$d_0^+ = \begin{cases} 0.56\left(\frac{r^+}{20}\right)^{2.5}; & \leqslant r^+ < 20 \\ 0.63\zeta(r^+) + 0.028r^+; & 20 \leqslant r^+ < 90, \\ 0.031r^+ - 0.27; & 90 \leqslant r^+ \end{cases}$$

where $\zeta(r^+) = \sin[\pi((r^+ - 20)/70)^{0.9}]$. It is easy to see that ω_w decreases with increasing r^+, which leads to increase of C_f.

Under transitionally rough conditions, Knopp et al. (2009) use a linear blending function

$$k_w = \frac{u_*^2}{\sqrt{C_\mu}} \min(1, r^+/90), \tag{1.6}$$

for the k boundary condition, while Seo (2004) retains (1.1).

1.3. Modeling for roughness effects on transition

A few recent studies propose roughness transition models. They are based on a data correlation for the momentum thickness Reynolds number at which transition starts. Its form is $Re_{\theta t-rough}$ as a function of $Re_{\theta t-smooth}$, surface roughness, and turbulence intensity. The correlation in Stripf et al. (2009a) depends on both the roughness height and density, while Boyle and Stripf (2009) propose a simpler formula, which only depends on the roughness height. The dimensionless roughness height used by the former is r/δ_*, rather than the more general, r_+, used in Eqs. (1.1), (1.6) and (1.5). Hence, in the next section, the model will be calibrated with the correlation proposed by Boyle and Stripf (2009),

$$Re_{\theta t-rough} = \frac{Re_{\theta t-smooth}}{1 + Tu^{-0.625}(0.05(r^+ - 5))^{1.25}}. \tag{1.7}$$

Here $Re_{\theta t-smooth}$ is the critical $Re_{\theta t}$ for smooth walls proposed by Mayle (1991),

$$Re_{\theta t-smooth} = 400Tu^{-0.625}. \tag{1.8}$$

Tu is the free-stream turbulence intensity at the transition onset location, and r^+ is the dimensionless roughness height defined in Section 1.2. The term $r^+ - 5$ implies that a surface roughness can be considered hydraulically smooth if r^+ is less than 5. Also note that, by this correlation, transition onset becomes independent of the local turbulence intensity at high r^+ values.

Herein, we extend the smooth wall, bypass transition model of Ge et al. (2014) and Durbin (2012) to account for the effect of wall roughness. Bypass transition skips the stage of Tollmien–Schilichting instability and is triggered by free-stream disturbances penetrating into the boundary layer and/or by surface roughness. The model of Ge et al. (2014) is based on the $k - \omega$ turbulence model and an intermittency transport equation. It uses only local variables and is tensorally invariant.

Inspired by the idea of the equivalent sand grain roughness and the *displacement of origin* approach for the roughness modification in fully turbulent flow, a displacement of origin method is developed for the intermittency equation. (However, the sink term in the intermittency equation needs a non-displacement type of modification.) Eqs. 1.4, 1.5 and 1.6 are chosen as the boundary conditions for k and ω on rough walls.

Previously, Dassler et al. (2010) proposed a very different type of extension of a smooth wall transition model, which is known as the $\gamma - Re_{\theta t}$ model (Langtry and Menter, 2009; Menter et al., 2006). A transport equation was added on for a 'roughness amplification', A_r, that serves as a transition onset criterion. The production term of the transport equation for $\widetilde{Re_{\theta t}}$ in the $\gamma - Re_{\theta t}$ model was modified by a function of A_r.

A more recent paper Elsner and Warzecha (2014) introduced the roughness transition correlation by Stripf et al. (2009a) into the $\gamma - Re_{\theta t}$ model. However, the integral quantity δ_* has to be calculated at each time step, and provided at each point of the grid, so this model is not based on local variables. Both Dassler et al. (2010) and Elsner and Warzecha (2014) used the $k - \omega$-SST model, and they chose Wilcox's roughness boundary condition for the fully turbulent boundary layer.

2. Formulation of the model

In this section, the details of the roughness modification will be presented and the rational will be provided. The modification consists of two steps: the first step is to add an effective displacement the origin, depending on the equivalent sand grain roughness height, to the wall distance. The data correlation (1.7) is used to calibrate the effective displacement. The second step is to modify