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Large-Eddy simulation of turbulent pipe flow of power-law fluids

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ABSTRACT

Fully developed turbulent flows of power-law fluids in a cylindrical stationary pipe are investigated numerically by the use of large eddy simulation (LES) for various power law index ($0.5 \leqslant n \leqslant 1.4$) at different Reynolds numbers ($4000 \leqslant Re_s \leqslant 12{,}000$). To validate the present computations, the predictions are compared to the results reported in the archival literature for laminar and turbulent flows. The LES predictions agree reasonably favourably with the findings of the literature. The log-region of the mean axial velocity profile expands with increasing Re_s and decreasing power-law index n. The predicted friction factor for $n \leqslant 1$ at $Re_s = 4000$ is slightly overestimated in comparison with Dodge and Metzner correlation, and is better interpolated by Gomes correlation. With increasing n the apparent viscosity increases close to the wall and decreases for $y^+ > 30$. This implies that the turbulent fluctuations develop and are more intense further from the wall when $n > 1$ and closer to the wall when $n < 1$. The influence of Re_s and n on the higher-order statistics (skewness and flatness) is analyzed. Visualizations of the instantaneous filtered velocity fields exhibit turbulent patterns which develop more as n increases.

1. Introduction

The turbulent flows of non-Newtonian fluids are of importance in mechanical and engineering fields. They are encountered in a variety of engineering applications, e.g. drilling hydraulics, sewage transport, processing of mineral oil and polymer products, blood flow in arteries, and applications involving relatively high heat transfer rates. While the turbulence theory, the mathematical models and the numerical methods are well-advanced for Newtonian fluids, those for non-Newtonian fluids are not as developed. Some attempts have been made to explore the effect of the power-law index and Reynolds number on the velocity distribution and turbulence statistics. Computational models for non-Newtonian fluids can help to bridge the gap in the existing literature, and can contribute to developing the general theories of the turbulent flows of non-Newtonian fluids.

Malin (1997) used a modified k–ϵ model (a low Reynolds number k–ϵ model extended to power law fluids) to calculate the frictional resistance and the velocity profile for fully developed laminar and turbulent flows in smooth-walled tubes. A modification of the viscous damping that improves the predictions for non-Newtonian fluids is proposed. The presented k–ϵ predictions are in fairly good agreement with experimental data for the turbulent friction and the mean velocity profiles at various generalized Reynolds numbers and different values of the power-law index n.

A more conventional and general k–ϵ model was developed by Kyoungchul and HongSun (2012) to analyze non-Newtonian fluid flows for more complex and various engineering problems. The modified k–ϵ model is based on the standard one with wall and damping functions including the drag reduction phenomenon. In order to validate their modified k–ϵ model, numerical simulations are performed for shear-thinning fluids, at different values of the flow index $0.4 \leqslant n \leqslant 1$. The predicted friction factors and mean axial velocity profiles agree well with the experimental results of literature (Dodge and Metzner, 1959; Escudier and Presti, 1996; Ptasinski et al., 2001), and this agreement is much better than with the standard k–ϵ model. The proposed model also agree well with Malin's power law model in the simulation of blood flow. The authors indicate that the computational time and computer resources of the modified k–ϵ model are reduced by about one third of those required by the low Reynolds number k–ϵ models for power-law fluid (including Malin's model). The authors point out that their turbulence model better predicts the behaviour of high power-law fluids.

Direct numeral simulation of turbulent pipe flows of shear-thinning fluids was carried out by Rudman et al. (2004) for $n = 0.5$, 0.69 and 0.75, using a spectral element-Fourier method, at a moderate Metzner–Reed Reynolds number ($Re_{MR} \simeq 3000$ and

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Nomenclature

C_d	coefficient of the dynamic model
D	pipe diameter (m)
f	mean friction factor, $f = 2\tau_w/(\rho U_b^2)$
f_{DM}	Dodge and Metzner (1959) correlation for friction factor
f_G	Gomes (1987) correlation for friction factor
$F(v_i')$	flatness factor, $F(v_i') = \frac{\langle v_i'^4 \rangle}{\langle v_i'^2 \rangle^2}$
K	consistency index (Pa s^n)
L_z	length of the computational domain (m)
n	power law index
q_i	generic notation for q_r, q_θ and q_z
q_r, q_θ, q_z	variables $q_r = r v_r$, $q_\theta = r v_\theta, q_z = v_z$
Re_b	Reynolds number based on bulk velocity, $Re_b = \frac{U_b D}{\nu}$
Re_{cr}	critical Reynolds number between laminar and turbulent flows, $Re_{cr} = 2100\frac{(4n+2)(5n+3)}{3(3n+1)^2}$
Re_{MR}	Metzner–Reed Reynolds number $Re_{MR} = \frac{8\rho U_b^{2-n} D^n}{K(6+2/n)^n}$
Re_s	simulation Reynolds number, $Re_s = \frac{\rho U_{cL}^{2-n} R^n}{K}$
Re_w	generalized Reynolds number, $Re_w = \frac{\rho U_\tau D}{\eta_w}$
r	dimensionless coordinate in the radial direction scaled by the pipe radius
R	pipe radius (m)
S_{ij}	strain rate tensor
$S(v_i')$	skewness factor, $S(v_i') = \frac{\langle v_i'^3 \rangle}{\langle v_i'^2 \rangle^{\frac{3}{2}}}$
u_i	generic notation for the dimensionless velocity components v_r, v_θ and v_z
U_b	bulk velocity (m/s)
U_{cL}	centreline axial velocity. For analytical laminar profile, $U_{cL} = \frac{(3n+1)U_b}{n+1}$
U^+	mean axial velocity in wall units, $U^+ = U/U_\tau$
U_τ	friction velocity, $U_\tau = (\tau_w/\rho)^{1/2}$
v_r, v_z, v_θ	dimensionless radial, axial and azimuthal velocity components
y^+	distance from the wall in wall units, $y^+ = (1-r)U_\tau/\nu$
z	dimensionless coordinate in the axial direction scaled by the pipe radius

Greek symbols

η_w	mean apparent viscosity at the wall (m^2 s^{-1})
η	apparent viscosity, $\eta = K\dot{\gamma}^{n-1}$
$\eta_{d,w}$	dimensionless apparent viscosity at the wall, $\eta_w/(\rho U_{cL} R)$
$\dot{\gamma}$	shear rate, $\dot{\gamma} = (2S_{ij}S_{ij})^{1/2}$
$\dot{\gamma}_{d,w}$	dimensionless shear rate at the wall, $\dot{\gamma}_{d,w} = \gamma_w R/U_{cL}$
θ	dimensionless angular coordinate in the circumferential direction
ν_t	turbulent viscosity
ρ	density (kg/m^3)
τ_{ij}	subgrid stress tensor, $\bar{\tau}_{ij} = -2\nu_t \bar{S}_{ij}$
τ_w	mean averaged fully-established wall shear stress, $\tau_w = \frac{D}{4}\frac{\partial p}{\partial z}$

Superscripts

$\langle(\cdot)\rangle$	statistically averaged
$(\cdot)^+$	normalized by u_τ or η_w
$\overline{(\cdot)}$	filtered variable
$(\cdot)'$	fluctuation component

Subscripts

c	centreline
L	laminar
w	wall

4000). A similar DNS study at a higher Metzner–Reed Reynolds number ($Re_{MR} = 7500$) was conducted by Rudman and Blackburn (2012). In the log-region, the velocity profile was shown to agree well with the experimental data by Rudman et al. (2001, 2002). The friction factors predicted by DNS were 10–15% higher than those referred to in earlier research (Dodge and Metzner correlations obtained from experiments). The authors reported that this is most likely related to the imperfect fit of the experimental data concerning fluids with power-law rheology. It was shown that, for a given Reynolds number, the flow deviates further from the Newtonian profile as the power-law index n decreases, and the results suggest that the transition to turbulence is delayed. Moreover, the shear-thinning or thickening rheologies did not result in major changes to the nature of the flow at $Re_{MR} = 7500$.

Direct numerical simulation (DNS) and large-eddy simulation (LES) are techniques well suited for predicting turbulent non-Newtonian fluid flows, because a detailed picture of the turbulent structures, profiles of turbulence energy, rms and Reynolds stresses are difficult to obtain experimentally. In DNS, numerically accurate and complete resolution of all spatial and temporal flow scales is required and no turbulence model is used. In LES, an accurate numerical resolution of a wide range of scales is required and only the smallest scales are modelled using a subgrid scale (sgs) turbulence model. While DNS is clearly a very useful tool for accurately simulating the turbulent flows, LES however can yield quantitatively accurate predictions at a computational cost which is significantly lower than the corresponding DNS one, since the effect of the smallest scales in LES is modelled and the mesh is relatively coarse. Moreover, when the Reynolds number is significant, LES provides an effective tool for predicting the effect of the flow index and Reynolds number on the turbulent fields of non-Newtonian fluids.

There are very few studies employing LES for non-Newtonian fluids. To predict the turbulence features in non-Newtonian fluid flows, Ohta and Miyashita (2014) developed a turbulence model that can reproduce the DNS results. They pointed out that constructing a new turbulence model extended for non-Newtonian fluids would obviously be difficult, since the new model would have to consider additional terms in the filtered Navier–Stokes equations (i.e. it can hardly be expected to construct a turbulence model by introducing turbulence corrections to the additional terms). Therefore, they considered a different approach: they proposed an extended Smagorinsky model with a correction for the filter width of the locally varying viscosity. Ohta and Miyashita (2014) performed DNS and LES of turbulent channel flow, in two non-Newtonian fluids with the viscosity described by both the power-law model (n = 0.85 and 1.15) and Casson's model. By performing LES with the Smagorinsky model as sgs model, extended according to the results of the DNS, they evaluated the reliability of the extended sgs model. They found that it could more accurately predict the velocity of turbulent flows of fluids described by both Casson's model and power-law model as compared to the standard Smagorinsky model (i.e. the results of LES with the extended model agree more with those obtained by DNS with high resolution). Consequently their study showed that the Smagorinsky model of non-Newtonian turbulent flows could be universally treated via a spatial scaling of the locally varying viscosity.

Thais et al. (2010) proposed an LES approach for viscoelastic turbulent channel flows, based upon a temporal deconvolution method (which was developed for LES of Newtonian channel