International Journal of Heat and Mass Transfer 100 (2016) 191–198

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# Determination of the relative significance of material parameters for concrete exposed to fire

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ARTICLE INFO

*Article history:*
Received 21 December 2015
Received in revised form 1 April 2016
Accepted 14 April 2016
Available online 7 May 2016

*Keywords:*
Concrete material parameters
Sensitivity analysis
Fire

ABSTRACT

When concrete is exposed to fire, spalling may occur. As regards this phenomenon, many numerical models have been proposed and developed during the last 30 years. It is state-of-the-art that the governing equations for consideration of spalling of concrete, which represent the basis of such models, are fully coupled, accounting for several important physical and chemical phenomena of different significance. The number of material parameters has increased with increasing complexity of these equations. This suggests exploring the relative significance of these parameters. For that purpose, a one-dimensional mathematical model for the analysis of concrete exposed to fire is used. By means of exploring the sensitivity of the temperature and the gas pressure at a specific point in a concrete floor slab to variations of several material parameters in the employed constitutive relations, the goal of this work is achieved.



## 1. Introduction

Spalling is the moderate or explosive breaking off of pieces from concrete subjected to high temperature. It leads to the reduction of the cross-sectional area and to the exposure of the steel reinforcement. This may eventually result in a significant reduction of the structural integrity of buildings. As for the spalling mechanism, two different theories have been proposed: the thermal stress theory [1–3] and the pore pressure theory [4,5]. Numerical models based on these two theories have been applied to predict spalling and to assess its consequences. The thermal stress theory concentrates on the stress variation due to the temperature gradient. Hence, the number of material parameters in the coupled numerical models is relatively small. The pore pressure theory considers the gas accumulation caused by mass and heat transport as the reason for spalling. This results in a larger number of material parameters in the respective numerical models. Therefore, the present investigation is based on the model as regards the pore pressure theory.

Governing equations for pore pressure models of different levels of complexity were developed by means of multi-field analysis of porous media. They range from the analysis of a single phase in the early age of the development of numerical models to multi-phase analysis, which has become state-of-the-art [5–10]. Most significant coupled phenomena, such as heat transportation, permeation and diffusion of mass as well as phase changes have been considered in numerical models.

The increasing complexity of the governing equations has resulted in an increase of the number of constitutive relations which contain the material parameters. Their function is to describe various concrete properties. Thus, they may have a great influence on the accuracy of the numerical results. Usually, they are either obtained from the literature or from experiments. Obviously, carrying out experiments is less economic. Experimental results, on the other hand, permit a more accurate description of the concrete. Considering the relatively large number of material parameters, it is important to explore their relative significance before choosing an appropriate way to obtain them.

Determination of the relative significance of some material parameters is documented in several papers. Gawin [11,12] discussed the relative significance of different flux items and explored several simplified cases which omit some physical phenomena, the conclusions of which can help us simplify the governing equations. Zhang [13] only analyzed the influence of porosity, permeability and saturation for concrete exposed to fire. Davie [14] also discussed the thermal conductivity apart from the aforementioned three parameters and his analysis is concerned with a pre-stressed concrete pressure vessel which is exposed to the slow heating process and the relatively low temperature. The heating duration is about 35 years and the highest temperature

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http://dx.doi.org/10.1016/j.ijheatmasstransfer.2016.04.042
0017-9310/

is about 90 °C. The results of the last two separate analyses are shown in the form of graphs, which renders a comparison of the significance of different parameters difficult. Besides, the number of the analyzed parameters is very small. Therefore, it is necessary to determine the relative significance of all the important material parameters.

In order to consider material parameters in a systematic manner, constitutive relations containing these parameters are classified in three groups according to their specific features. Since the temperature and the gas pressure in the concrete are decisive factors for spalling of concrete, the goal of this work is to investigate the sensitivity of these two physical quantities to variations of the material parameters. Numerical results from a reference example, based on a coupled mathematical model, are presented. They are compared with the ones that are obtained by changing the value of only one parameter each before carrying out the next analysis. The relative significance of the material parameters is demonstrated in the form of percentage differences.

The paper is organized as follows: In Section 2, a one-dimensional analysis model is introduced. It contains governing equations that are applicable to coupled multi-field problems and constitutive relations containing the material parameters. Section 3 contains basic material parameters and a numerical reference example concerning a floor slab. In Section 4, named sensitivity analysis, the relative significance of the material parameters in the different groups is quantified. Concluding remarks are made in Section 5.

## 2. Mathematical model

The mathematical model adopted herein was originally proposed by Gawin et al. [10,15]. It considers the major physical and chemical phenomena in concrete exposed to fire and includes more material parameters than other models. Since the paper focuses on the sensitivity of two important quantities of the concrete to variations of material parameters, a one-dimensional analysis model is considered to be adequate. It is based on the aforementioned mathematical model by Gawin et al., which was simplified by Zeiml [16].

### 2.1. Governing equations

In the model, concrete is treated as a multi-phase medium, comprising a solid skeleton, water and gas (vapor and dry air). Based on the mass and enthalpy balance equations and introducing relevant basic physical laws, i.e. Darcy's law and Fick's law, the governing equations of the model, with the capillary pressure $p^c$, the gas pressure $p^g$, and the temperature $T$ as unknowns, are given as follows [16]:

*mass balance equation for the dry gas phase:*

$$\begin{aligned}&-n\rho^{ga}\tfrac{D^s S_w}{Dt}+n(1-S_w)\tfrac{D^s\rho^{ga}}{Dt}-\rho^{ga}(1-n)(1-S_w)\beta_s\tfrac{D^sT}{Dt}-\mathrm{div}\left(\rho^{ga}\tfrac{\mathbf{k}k^{rg}}{\eta^g}\mathrm{grad}p^g\right)\\&-\mathrm{div}\left[\rho^g\tfrac{M_aM_w}{M_g^2}\mathbf{D}_{eff}\mathrm{grad}\left(\tfrac{p^{ga}}{p^g}\right)\right]=-\tfrac{(1-n)\rho^{ga}(1-S_w)}{\rho^s}\tfrac{\partial\rho^s}{\partial\xi}\tfrac{D^s\xi}{Dt}-\rho^{ga}(1-S_w)\tfrac{\dot m_{dehydr}}{\rho^s};\end{aligned}\tag{1}$$

*mass balance equation for the water phase:*

$$\begin{aligned}&n(\rho^w-\rho^{gw})\tfrac{D^sS_w}{Dt}+n(1-S_w)\tfrac{D^s\rho^{gw}}{Dt}+nS_w\tfrac{D^s\rho^w}{Dt}-\mathrm{div}\left(\rho^{gw}\tfrac{\mathbf{k}k^{rg}}{\eta^g}\mathrm{grad}p^g\right)\\&-(1-n)\beta_s[\rho^{gw}+(\rho^w-\rho^{gw})S_w]\tfrac{D^sT}{Dt}-\mathrm{div}\left(\rho^w\tfrac{\mathbf{k}k^{rw}}{\eta^w}\mathrm{grad}p^w\right)\\&-\mathrm{div}\left[\rho^g\tfrac{M_aM_w}{M_g^2}\mathbf{D}_{eff}\mathrm{grad}\left(\tfrac{p^{gw}}{p^g}\right)\right]=-\tfrac{(1-n)[\rho^{gw}(1-S_w)+\rho^wS_w]}{\rho^s}\tfrac{\partial\rho^s}{\partial\xi}\tfrac{D^s\xi}{Dt}\\&-[\rho^{gw}(1-S_w)+\rho^wS_w]\tfrac{\dot m_{dehydr}}{\rho^s}+\dot m_{dehydr};\end{aligned}\tag{2}$$

*energy balance equation:*

$$\begin{aligned}&(\rho c_p)_{eff}\tfrac{\partial T}{\partial t}-(\rho^gc_p^g\tfrac{\mathbf{k}k^{rg}}{\eta^g}\mathrm{grad}p^g+\rho^wc_p^w\tfrac{\mathbf{k}k^{rw}}{\eta^w}\mathrm{grad}p^w)\mathrm{grad}T\\&-\mathrm{div}(\lambda_{eff}\mathrm{grad}T)=-\dot m_{vap}h-\dot m_{dehydr}l_\xi\end{aligned},\tag{3}$$

where $\dot m_{vap}$ is determined from

$$\begin{aligned}\dot m_{vap}=&-n\rho^w\tfrac{D^sS_w}{Dt}-nS_w\tfrac{D^s\rho^w}{Dt}+\rho^w(1-n)S_w\beta_s\tfrac{D^sT}{Dt}+div(\rho^w\tfrac{\mathbf{k}k^{rw}}{\eta^w}\mathrm{grad}p^w)\\&-\tfrac{(1-n)\rho^wS_w}{\rho^s}\tfrac{\partial\rho^s}{\partial\xi}\tfrac{D^s\xi}{Dt}+\dot m_{dehydr}-\rho^wS_w\tfrac{\dot m_{dehydr}}{\rho^s}.\end{aligned}\tag{4}$$

The mathematical symbols appearing in Eqs. (1)–(4) and in relations yet to be presented are defined in the Appendix A.

### 2.2. Constitutive relations

The constitutive relations describe physical and chemical properties of the individual phases inside the concrete. They are mainly based on experiments and on fundamental physical laws. These relations can be classified in three groups in terms of the process of their formulation and the characteristics of the respective expressions.

#### 2.2.1. Initial value group

Any constitutive relation in this group contains the initial value of the material parameter, expressed by this relation. The variation of this initial value reflects the relative significance of the considered material parameter for the analysis of concrete exposed to fire. This group includes the porosity $n$, the density $\rho^s$, the specific heat capacity of concrete, $c_p^s$, the thermal conductivity $\lambda^s$, the intrinsic permeability $k_{\mathrm{int}}$, and the saturation $S_w$.

The porosity depends on the temperature [17], as is the case with the density, the specific heat capacity, and the thermal conductivity [18]. The structure of the formulae for these quantities is the same. The formulae are given as

$$n=n_0+A_n(T-T_0),\tag{5}$$

$$\rho^s=\rho^{s0}+A_{\rho^s}(T-T_0),\tag{6}$$

$$c_p^s=c_p^{s0}+A_{c_p^s}(T-T_0)\qquad(\text{for } T\leqslant 800^\circ\mathrm{C}),\tag{7}$$

$$\lambda^s=\lambda^{s0}+A_\lambda(T-T_0)\qquad(\text{for } T\leqslant 374.15^\circ\mathrm{C}),\tag{8}$$

where $n_0$, $\rho^{s0}$, $c_p^{s0}$, and $\lambda^{s0}$ are initial values of $n$, $\rho^s$, $c_p^s$, and $\lambda^s$, respectively, at the reference temperature $T_0$. $A_n$, $A_{\rho^s}$, $A_{c_p^s}$, and $A_\lambda$ are empirical parameters, depending on the design of the concrete mix.

The intrinsic permeability $k_{\mathrm{int}}$ is mainly influenced by the temperature. Several formulae have been proposed for this quantity [8,9,19]. In the present model, a formula suggested by Zeiml et al. [20,21], is applied. It is given as

$$k_{\mathrm{int}}=k_{\mathrm{int},0}10^{A_k(T-T_0)},\tag{9}$$

where $k_{\mathrm{int},0}$ is the initial intrinsic permeability at $T_0$ and $A_k$ is a parameter obtained from experimental data.

The saturation $S_w$ is related to the temperature and the capillary pressure. Zeiml [16] proposed the following expression for this physical quantity:

$$S_w=\left[1+\left(\frac{E_s}{A_s}p^c\right)^{\left(\frac{1}{1-m}\right)}\right]^{-m},\tag{10}$$

where $E_s$ and $A_s$ are related to the temperature and $m$ is an empirical coefficient. The initial value of the saturation mainly depends on the capillary pressure at $T_0$.