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A ghost-cell immersed boundary method for simulations of heat transfer in compressible flows under different boundary conditions

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ABSTRACT

In this paper we describe the implementation of a ghost-cell immersed boundary method for compressible flow with Dirichlet, Neumann and Robin boundary conditions. A general second-order reconstruction scheme is proposed to enforce the boundary conditions via ghost points. The convergence test shows that the present method has a second-order accuracy for three types of boundary conditions. Laminar flow heat transfer problems are used to test the capability of the present method to handle different boundary conditions with stationary and moving boundaries. The compressible effect on the heat transfer process is then studied to illustrate the advantage and necessity of combining IB methods with a compressible flow solver.

1. Introduction

The development of accurate and efficient methods for arbitrarily complex geometries and multiple boundary conditions has been one of the main issues in computational fluid dynamics. The immersed boundary (IB) method has been demonstrated to have the capability of handling complex fluid–structure interaction problems with high efficiency. The advantages of the IB method, such as simplicity in grid generation, savings in computer resources and straightforward parallelization, have expanded its applications in multiphase flow simulations.

The immersed boundary method was first introduced by Peskin to simulate the blood flow around a human heart valve [1]. The main idea of this method is to use a Cartesian grid for fluid flow simulation together with a Lagrangian representation of the immersed boundary. A forcing term is introduced to represent the interaction between the immersed boundary and the fluid, and a discrete Dirac-delta function is used to smooth this singular force on the Euler grid [2]. Since then, numerous modifications and improvements have been made, which are well discussed and categorized in [3–5].

The idea of the ghost cell immersed boundary (GCIB) method is based on the work of Mohd-Yusof [6] and Fadlun et al. [7]. The GCIB method treats the immersed boundary as a sharp interface, and does not require the explicit addition of discrete forces in the governing equations, thus it can be easily combined with the existing solvers. The boundary condition on the IB is enforced through the "ghost cells". The variable values of the ghost cells are calculated with the IB boundary conditions and the fluid variables near the boundary. The flow solver senses the presence of the immersed boundary through the extrapolated values at the ghost points [8]. In order to avoid numerical instability caused by the large, negative weighting coefficients in the extrapolation formula, the concept of mirror points lying inside the flow domain is adopted to ensure suitable weighting coefficients in the reconstruction formula. Different interpolation procedures for the mirror point [9] and extrapolation procedures for the ghost point [10,11] can be utilized to obtain a second or even higher order accuracy [12–14]. The GCIB method has shown large potential to handle different fluid–solid interaction problems, including those involving highly complex geometries [15–17] and moving/deforming objects [18–20].

Extension of the immersed boundary method to heat transfer problems has gained its popularity since Kim and Choi [21]. Many efforts have been made to improve the accuracy of thermal boundary condition enforcement and broaden its application. Dirichlet and Neumann type boundary conditions for IB methods have been studied by many researchers [21–26]. While for more complicated boundary conditions, such as Robin and conjugate boundary conditions, the number of available studies are still limited [27–29]. The Robin boundary condition, also known as the mixed Dirichlet–Neumann boundary condition, is important in heat and mass diffusion processes coupled with convection and has been

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used for prescribing thermal or mass fluxes and surface reactions [30–33].

Many fluid dynamic problems of engineering interest involve compressible flows with very different Mach numbers and complex heat/mass transfer processes. There are plenty of parameters that are strongly coupled to each other like density or temperature in a compressible flow. The influence of temperature ratio and Mach number on the flow region has been illustrated by Wang et al. [34] and Sabanca et al. [35]. These results indicate that there is a large difference in the heat transfer phenomena between the compressible flow and the incompressible one, and show the limitation of the incompressible solver for flows with large temperature ratios and high Mach numbers. So far, only a few IB methods are designed for compressible flows [36,37]. Therefore, it is desirable to develop an IB method based on a compressible solver for heat transfer simulation with complex boundary conditions, which is more practical and has a potential to solve chemical reaction problems.

To this end, a general boundary condition treatment, using the ghost-cell immersed boundary method for compressible flows, is developed and validated in the present work. The interaction between immersed bodies and the fluid is expressed by ghost points inside the immersed bodies, and these ghost points ensure that boundary conditions are satisfied precisely on the immersed boundary. Different reconstruction stencils are carried out to maintain the second-order accuracy of the method for different boundary conditions.

The reminder of the present paper is organized as follows. Sections 2 and 3 describe the numerical methodology including the flow solver and the ghost-cell immersed boundary method. In Section 4 the capability of the proposed methodology to handle heat transfer problems with different boundary conditions in compressible flows is verified and validated, including flows with moving interface and medium Mach numbers. Section 5 is devoted to summary and conclusions.

2. Governing equations

The Navier–Stokes equations for a compressible fluid are introduced here. The continuity equation is solved in the form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \quad (1)$$

where ρ is the fluid density, $\boldsymbol{u}$ is the fluid velocity, t is time.

The momentum equation is written in the form

$$\frac{D\boldsymbol{u}}{Dt} = \frac{1}{\rho}(-\nabla p + F_{vs}), \quad (2)$$

where p is the pressure, $F_{vs} = \nabla \cdot (2\rho \upsilon \boldsymbol{S})$ is the viscous force, υ is the kinematic viscosity, $S_{ij} = \frac{1}{2}(\partial u_i/\partial x_j + \partial u_j/\partial x_i) - \frac{1}{3}\delta_{ij}\nabla \cdot \boldsymbol{u}$ is the trace-less rate of strain tensor and $D/Dt = \partial/\partial t + \boldsymbol{u} \cdot \nabla$ is the convective derivative.

The energy equation is

$$\frac{\partial \ln T}{\partial t} = -\boldsymbol{u} \cdot \nabla \ln T + \frac{1}{\rho c_p T}(\nabla \cdot (k \nabla T) + 2\rho \upsilon \boldsymbol{S} \otimes \boldsymbol{S}), \quad (3)$$

where T is the temperature, c_p is the specific heat at constant pressure and k is the heat conductivity.

The ideal gas equation of state is given by

$$p = \rho R T \quad (4)$$

and can be reduce to

$$p = c_s^2 \rho. \quad (5)$$

for isothermal flow. Here $c_s = \sqrt{\partial p/\partial \rho}$ is the speed of sound.

The solvers of the PENCIL CODE [38] are utilized for the present study. The sixth-order centered finite-difference scheme for spatial derivatives and third-order Runge–Kutta scheme for time advancement are used to solve the above governing equations. In the simulations, the time step is specified as the Courant time step that is calculated based on a number of constraints involving maximum values of velocity, viscosity, and other quantities on the right hand sides of the evolution equations.

3. Ghost-cell compressible immersed boundary (GCCIB) method

In order to impose the boundary condition in such a way that ensures a sharp interface separating the compressible fluid and the solid, a ghost-cell immersed boundary methodology is developed here. The advantage of easy implementation of this method enables us to use the existing solver of the PENCIL CODE. The basic idea of the GCCIB method developed here to handle different types of boundary conditions is based on the work of Haugen et al. [39].

A schematic diagram of the present GCCIB method is shown in Fig. 1. The domain in shadow denotes the solid domain and the rest is the fluid domain. For the sixth-order finite central difference scheme used here, three layers of ghost points (○) are needed to complete the discretization stencils near the boundary. The other grid points inside the solid domain are solid points (■) which are not used in the calculation. At the beginning of the simulations, a detection of the immersed boundary and assignments of ghost points and fluid points are carried out. Then the wall normal direction from each ghost point can be determined. In this study, the mirror points are defined as the points that are normal to the immersed boundary, lying in the fluid domain and have the same distance to the immersed boundary as their corresponding ghost points.

In most situations, the mirror points do not coincide with the grid points. Thus a bilinear interpolation for 2D cases (or tri-linear interpolation for 3D cases) is used to calculate the fluid properties at the mirror points. The bilinear interpolation for a mirror point with four surrounding fluid points can be expressed as

$$\phi(x, y, z) = C_1 xy + C_2 x + C_3 y + C_4. \quad (6)$$

Here ϕ denotes a generic variable at the mirror point. The four unknown coefficients can be determined using the variable values of the four surrounding points

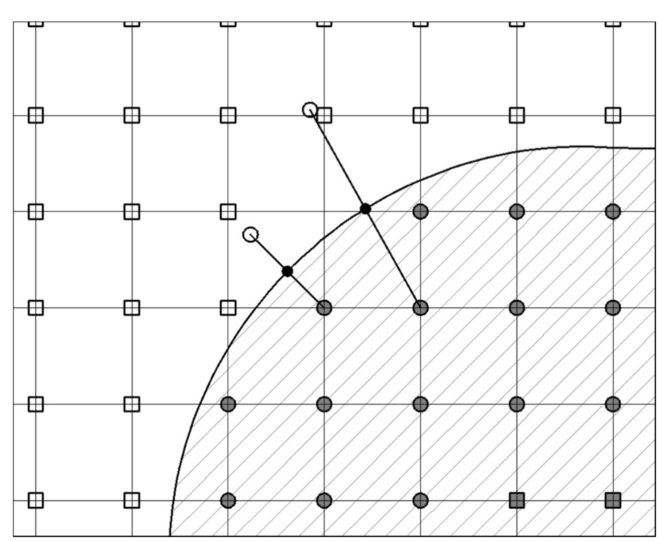

Fig. 1. 2D schematic diagram for the GCIB method, ghost points (◎), mirror points (○), boundary intersection (BI) points (●), fluid points (□) and solid points (■).