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Asymptotic accelerated boundary layer over the permeable wall

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ABSTRACT

The paper reports on numerical and analytical investigation of the laminar boundary layer with a favorable pressure gradient over a permeable wall. Researchers have obtained an analytical solution of boundary layer equations for asymptotic flow conditions. This solution allows proposing a relative asymptotic skin-friction function, which determines a degree of influence of flow acceleration and permeable wall on the flow. There are ranges of this function, where effects of permeable wall and streamwise pressure gradient have to be considered only in combination. Numerical simulation has showed that such combined influence of favorable pressure gradient and permeable wall extends the asymptotic flow. The study of a strong gas blowing into the accelerated flow has revealed that favorable pressure gradient impedes separation of the boundary layer. At that asymptotic flow starts from the point, where the separation would occur at gas injection into the zero-pressure gradient flow.

1. Introduction

Accelerated flows and streams over permeable surfaces take place in various technical systems. For example, gas injection (suction) is used to increase energy separation effectiveness [1–4]. Authors of some investigations [5,6] demonstrate that favorable pressure gradient decreases a recovery factor to improve energy separation as well. Flow acceleration can both increase the effectiveness of surface film cooling [7] and decrease surface protection [8].

Kays with some research groups carried out the most detailed and regular experiments with accelerated flows over the permeable surfaces [9]. These investigations showed that gas injection into the flow with favorable pressure gradient decreases skin-friction coefficient, and gas suction in contrast increases it. Most of experiments were realized for the turbulent regime. At the same time, Kays [10] demonstrated that turbulence is suppressed fully, and flow turns to the laminar regime at $K > 3.55 \times 10^{-6}$. Bourassa and Thomas [11] proved this critical value of acceleration parameter in their experiments as well. So, the laminar regime of the flow can exist in a wide range of acceleration parameters and intensity of gas suction (injection).

Following Kays and Moffat [9], we use the definition of asymptotic boundary layer as the flow with “a state of equilibrium for which Re** is constant”. One of such flows is the well-known asymptotic suction boundary layer, when $c_f = |2\overline{j_w}|$. We discuss conditions of asymptotic flow existence in the Section 3.2.

To integrate Falkner – Scan equation, Koh and Hartnett [12] showed that suction from the laminar flow over the porous wedge decreases hydrodynamic thickness of the boundary layer, leads to the more filled streamwise velocity profiles, and, as a consequence, increases skin-friction coefficient. Besides, skin-friction coefficient grows with expansion of wedge angle independently of suction intensity.

Literature overview has shown that a problem of flows with the simultaneous action of favorable pressure gradient and permeable wall is an absence of a single parameter, characterizing the influence of these factors on the flow. To generalize flow parameters Kays et al. [13] propose the relation $\overline{j_w}/\sqrt{K}$, where $\overline{j_w}$ is the relative intensity of gas suction (injection), and K is the acceleration parameter. However, the authors themselves use this relation rather poorly, and other investigations do not refer to it.

There is no analytical expression for the velocity and skin-friction coefficient in accelerated flows over permeable wall even for the laminar regime. Pohlhausen obtained an analytical solution of the momentum equation for the sink flow in transformed coordinates [14–16]. At that the derived ordinary momentum equation has the analytical solution only for the case of impermeable wall [13].

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Nomenclature

c_f skin-friction coefficient
$\overline{j_w} = V_w/U_e$ intensity of gas injection or suction
$K = (\nu/U_e^2)dU_e/dx$ stream acceleration parameter
L length of wall [m]
P pressure [Pa]
Re_x Reynolds number based on streamwise coordinate x
Re_y wall-distance Reynolds number based y-coordinate and main flow velocity U_e
Re_δ hydrodynamic thickness Reynolds number
Re^* displacement thickness Reynolds number
Re^{**} momentum thickness Reynolds number
U, V velocity components in the x, y directions, respectively [m/s]
x, y streamwise and wall-normal coordinates relative to surface of streamlined body [m]

Greek symbols
α convergence angle of top plane of channel [°]
δ thickness of hydrodynamic boundary layer [m], $U/U_e = 0.995$
$\delta^* = \int_0^\infty (1 - U/U_e)dy$ displacement thickness [m]
$\delta^{**} = \int_0^\infty U/U_e(1 - U/U_e)dy$ momentum thickness [m]
ν kinematic viscosity [m^2/s]
Ψ_{aw} relative asymptotic skin-friction function

Subscripts
0 flow quantities at the start of flow
e flow quantities in external flow
w parameter at the wall

The momentum equation has the other analytical solution for the asymptotic suction from the zero-pressure gradient flow [16]. This solution is based on a hypothesis about a negative value of the wall-normal velocity constant across the boundary layer and equal to the magnitude at the wall V_w. On the basis of results of numerical simulation in the paper [17] we assumed that wall-normal velocity has a linear distribution across the boundary layer with the asymptotic favorable pressure gradient: $V/U_e = -K\,\mathrm{Re}_y$ and $\mathrm{Re}_y = U_e y/\nu$. As a result, we obtained the analytical expression for the streamwise velocity and derived a simple formula for the skin-friction coefficient $c_f = \sqrt{2\pi K}$. Pohlhausen obtained the similar formula using his method [14].

The recent article presents an analytical solution of the momentum equation under the simultaneous presence of favorable pressure gradient and permeable wall. The solution is based on the hypothesis about wall-normal velocity distribution similarly to the above described flows with asymptotic suction and with asymptotic favorable pressure gradient. We investigated numerically the applicability limits of the obtained expressions and the influence of favorable pressure gradient on the laminar boundary layer separation at gas blowing.

2. Flow configuration and simulation approach

We consider air flow in a plane convergent channel, where the acceleration parameter $K = (\nu/U_e^2)dU_e/dx$ remains constant over the entire channel length (Fig. 1). This is necessary for the existence of asymptotic flow region. As pointed in the Fig. 1, K-parameter can be determined by the inclination angle α of top wall, the initial velocity U_0 and the initial height of channel h_0. The lower wall is permeable with a constant wall-normal air flux $\overline{j_w} = V_w/U_e$, where negative values correspond to gas suction and positive ones to air injection. The sloped top wall of the channel is assumed to be located sufficiently far from the lower channel wall, i.e. $h_0 >> \delta$, so that the analysis can be focused on the accelerated hydrodynamic boundary layer developing over the bottom surface. The accelerated main flow and injected gas have the same properties at temperature of 300 K and atmospheric pressure.

2.1. Equations and boundary conditions

The flow considered can be well approximated by the parabolized two-dimensional momentum and continuity equations for a steady laminar incompressible boundary layer:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{dP}{dx} + \nu\frac{\partial^2 U}{\partial y^2}, \tag{1}$$

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0. \tag{2}$$

At the wall, we set the no-slip condition and intensity of gas suction (injection):

$$y = 0: \; U = 0, \quad V = V_w, \quad V_w = \overline{j_w} \cdot U_e, \quad \overline{j_w} = const. \tag{3}$$

At the outer edge of the boundary layer, the velocity U_e was determined from the integration of the acceleration parameter $K = (\mu/\rho U_e^2)dU_e/dx$ with constant viscosity of air and initial velocity U_0:

$$U_e = -(Kx/\nu - 1/U_0)^{-1}. \tag{4}$$

The initial velocity was defined by $U_0 = aM_{e\,\max}/(1 + K\mathrm{Re}_{x\,\max})$, where a is the acoustic velocity, $M_{e\,\max}$ and $\mathrm{Re}_{x\,\max}$ are the maximum Mach and Reynolds numbers, assumed for all cases to keep the same values, $M_{e\,\max} = 0.4$ and $\mathrm{Re}_{x\,\max} = 10^8$.

2.2. Numerical method

The system of differential Eqs. (1) and (2) was solved by a finite difference method using a fully implicit scheme with accuracy of $o(\Delta x) + o(\Delta y)^2$ [18]. The calculations were performed for a computational domain of 14 m in the streamwise (x) and 0.2 m in the wall-normal (y) direction using a non-uniform orthogonal grid $N_x \times N_y = 118{,}000 \times 400$. The criterion for convergence was

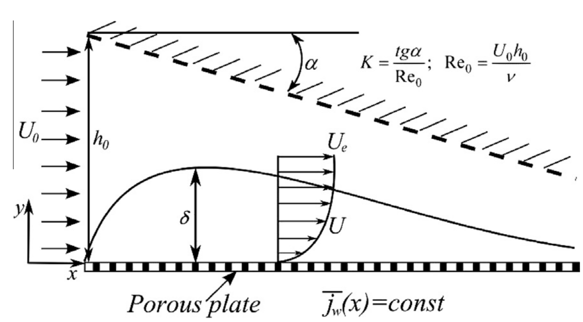

Fig. 1. Schematics of the flow considered.