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# A moving boundary model for two-phase flow heat exchanger incorporated with relative velocities between boundaries and fluid

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ABSTRACT

Moving Boundary (MB) dynamic model is an appealing approach for investigation of advanced control schemes for two-phase flow heat exchanger. For the confusion of relative velocities between boundaries and fluid existing in the previous MB model, this paper presents a modified moving boundary model. The dynamic model incorporated with the relative velocities is derived from physical principles of mass and energy conservation. And the model is then implemented in a novel underwater HYDROX system to predict cyclic performance. The simulation results from discretized model using MATLAB language show that the oscillations which is known as "*Chattering*" have been suppressed.



## 1. Introduction

Current heat exchanger (evaporator or condenser) system models are more appropriate for advanced control strategies. Bendapudi and Braun [1] discussed such modeling approaches including both finite volume model and Moving Boundary (MB) model. MB model has been proved much faster by Grald and MacArthur [2], which makes it to be the first choice for control system design [3].

The idea of MB model is to dynamically track the length of different regions in heat exchanger. And then, mass and energy conservation equations are formulated for each Control Volumes (CVs) with variable boundaries. A simple geometry for MB in an evaporator can be seen in Fig. 1.

Several works have described MB dynamic models. Adams et al. [4] pioneered MB models. Ray and Bowman [5] developed a nonlinear model based on the work of Adams. They described a three-region model with time-varying phase boundaries by a set of nonlinear differential and algebraic equations derived from the fundamental equations of conservation of mass, momentum, and energy. Extensions of this work for solar applications are presented in [6,7]. Dhar and Soedel [8] employed a simplified heat exchanger model in which spatial dependency was ignored. Mckinley and Alleyne [9], Mancini [10], Rasmussen [11] and Bonilla et al. [12–14] have presented the remarkable MB model reviews for two-phase flows.

He et al. [15,16] was the first to suggest the use of MB models for multi-input multi-output (MIMO) control design. He presented linearized two-region MB models for the evaporator and condenser with adequate validation. Wei et al. [17] presented MB model approach, separately for refrigeration systems and organic Rankine cycles, extending the work of He et al. [15,16].

Using first principles of mass and energy conservation, along with three separate MB formulations, Jensen [18], Jensen and Tummescheit [19] have presented a general moving boundary model for the conceivable cases of flow, including equations governing radiative, conductive and convective losses. Yebra et al. [20] extended the MB formulation from [15] including the momentum balance equation discretized in CV by the finite volume method in order to account for pressure drop. Li and Alleyne [21] developed a switching evaporator model, extending the condenser model previously developed in [9]. Also, experimental validation was presented by considering two test cases. Gräber et al. [22] derived their MB models in an elegant way from first principles, and proposed a validation procedure based on infrared thermography. Cecchinato and Mancini [23] presented a generalized intrinsically mass conservative evaporator model based on the MB approach. Zapata et al. [24] introduced a dynamic model of a once-through-to-superheat solar steam receiver for electricity generation following the approach presented by Mckinley and Alleyne [9] for refrigeration systems and Wei et al. [17] for Organic Rankine cycles.

An essential aspect of the MB formulation is pressure drop throughout the heat exchanger. Most researchers above assumed the pressure drop in the heat exchanger to have a negligible effect

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**Nomenclature**

| | |
|---|---|
| $A$ | area ($m^2$) |
| $\rho$ | density ($kg/m^3$) |
| $D$ | diameter (m) |
| $h$ | enthalpy (J/kg) |
| $q$ | heat flux per unit length (W/m) |
| $Q$ | heat (J) |
| $L$ | length (m) |
| $B_{nz}$ | nozzle pressure ratio |
| $Nu$ | Nusselt number |
| $S$ | perimeter (m) |
| $k$ | ratio of specific heat |
| $c$ | specific heat capacity (J/(kg K)) |
| $T$ | temperature (K) |
| $\lambda$ | thermal conductivity (W/m K) |
| $v$ | velocity (m/s) |
| $V$ | capacity ($m^3$) |
| $\varepsilon$ | degree of partial admission |
| $\eta$ | dynamic viscosity (Pa s) |
| $R_g$ | gas constant (J/(mol K)) |
| $\alpha$ | heat transfer coefficient (J/(kg m)) |
| $\nu$ | kinematic viscosity ($m^2/s$) |
| $m$ | mass (kg) |
| $\alpha_{nz}$ | nozzle angle (rad) |
| $Pr$ | Prandtl number |
| $P$ | pressure (Pa) |
| $Re$ | Reynolds number |
| $u$ | special internal energy (J/kg) |
| $t$ | time (s) |
| $u_{trb}$ | turbine linear velocity (m/s) |
| $P_u$ | wheel power (W) |

*Subscripts*

| | |
|---|---|
| $amb$ | ambient |
| $cmb$ | combustor |
| $s$ | design condition |
| $g$ | gas phase |
| $i$ | inner wall |
| $l$ | liquid phase |
| $o$ | outer wall |
| $bo$ | outlet boundary |
| $rw$ | reagent water |
| $sa$ | saturation |
| $trb$ | turbine |
| $av$ | average |
| $cnd$ | condenser |
| $evp$ | evaporator |
| $in$ | inlet |
| $bi$ | inlet boundary |
| $nz$ | nozzle |
| $out$ | outlet |
| $rdu$ | reactor |
| $wall$ | reactor wall |
| $w$ | tube wall |
| $e$ | two-phase |

on the dynamic response and they calculate thermodynamic properties from the predicted pressure. The exception are Adams et al. [4], Ray and Bowman [5], Yebra et al. [20] and Tian et al. [25,26], where static pressure drop was included. But the lumped thermodynamic properties were still evaluated at isobaric conditions. Also, all researchers above neglected any changes in kinetic energy.

This article uses first principles of mass and energy conservation to produce a modified MB formulation. Differences between this model and previous work are: an explicit inclusion of relative velocities between fluid and boundaries and consideration of pressure loss and kinetic energy. The effects of including these modifications relative to past models are discussed, and this model is shown to be suitable for the development of advanced control algorithms.

## 2. Previous models

According to the literature [12–20], the general governing equations for the time-dependent equations for conservation laws, applying the nomenclature in Fig. 2, is presented as Eqs. (1) and (2).

Mass balance:

$$A\frac{d}{dt}\int_{z_{in}}^{z_{out}}\rho dz + \rho_{in}A\frac{dz_{in}}{dt} - \rho_{out}A\frac{dz_{out}}{dt} = \dot{m}_{in} - \dot{m}_{out} \tag{1}$$

Energy balance:

$$A\frac{d}{dt}\int_{z_{in}}^{z_{out}}\rho h dz - AL\frac{dp}{dt} + A\rho_{in}h_{in}\frac{dz_{in}}{dt} - A\rho_{out}h_{out}\frac{dz_{out}}{dt} = \dot{m}_{in}h_{in} - \dot{m}_{out}h_{out} + qL \tag{2}$$

where, $z_{in}$ and $z_{out}$ are the locations of the inlet and outlet boundaries; $dz_{in}/dt$ and $dz_{out}/dt$ are the velocities of the boundaries. The "Inlet" and "Outlet" refer to the case of an evaporator and are switched in the case of a condenser.

In the mass balance of Eq. (1), the first term on the left hand side describes the rate of mass change in the CV caused by density change. The second and third terms describe the mass change in the CV due to changes in boundary positions. The two terms on the right hand side are the mass flows through the inlet and outlet boundaries.

In the energy balance of Eq. (2), the first term on the left hand side is the change rate of enthalpy in the CV, the second term is a consequence of using enthalpy and not internal energy in the first term. The third and fourth terms account for the change of enthalpy due to changes in boundary locations. The two first terms on the right hand side are the convective enthalpy through the boundaries, and the last term is the heat flow from the tube wall.

The mass balance shown in Eq. (1), can be physically interpreted that the changes of mass flow rate equal to the sum of

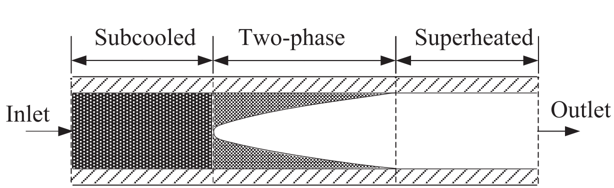


**Fig. 1.** Schematic of MB for an evaporator.

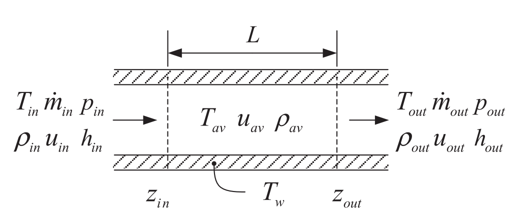


**Fig. 2.** Schematic of a CV.