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Fractal analysis of permeability of dual-porosity media embedded with random fractures

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ABSTRACT

Dual-porosity media embedded with random fractures widely exist in water and oil reservoirs, soils etc. The permeability of such media has been one of focuses in the area of mass transfer in porous media in the past decades. In this paper, analytical scheme for permeability of the media is presented by the fractal theory and technique based on the fact that the length distribution of random fractures obeys the fractal scaling law and flow in each fracture follows the cubic law. The porous matrix of the media is assumed to consist of a bundle of tortuous capillaries whose sizes also follow the fractal scaling law. The analytical expression for permeability is derived and is found to be a function of the fractal dimensions and the microstructural parameters of the media. The present results show that the proposed permeability model can reveal more mechanisms of seepage characteristics in the media than the traditional models.

1. Introduction

Fractures are usually embedded in an isotropic porous medium (often called matrix medium) to form a so-called dual-porosity medium, which was first proposed by Barrenblatt et al. in 1960 [1] and Warren and Root in 1963 [2]. Since then, models and transport properties in the media have received the steady concern in the past decades. The dual-porosity media embedded with random fractures widely exist in nature such as the cracked soil, fractured rock, and water/oil/gas reservoirs etc.

The dual-porosity media are composed of porous matrix and fracture networks, each with its own properties. Usually, the matrix permeability is much lower than that of fracture networks. However, the matrix has the great ability to store fluid, while the fracture networks play an important role in transport liquid. To characterize the transport properties in such media, Darcy's law is often used to describe the seepage characters in fracture networks and matrix medium. The matrix pore size and distribution, and fracture porosity, aperture, length, density and orientation usually have the significant influences on permeability of the media. In earlier studies, researchers set up dual-porosity medium models about matrix pore distribution and fracture networks by assuming that the media have ideal structures such as the parallel fracture networks [3], the orthogonal plane network cracks [4,5], alternate level matrix layer and fractures [6], orthogonal fracture cutting matrix rock model for hexahedron [2,7], etc. However, these ideal models exhibit larger deviations from actual situations.

Then, Van [8] considered the number of fractures and angles between flow directions and fractures in his model. He proposed a dual-porosity medium model and obtained an expression for permeability, i.e. $K_t = K_m + \sum_{i=1}^{n} K_{fi} \cos^2\theta$, where K_t and K_m are respectively the permeabilities of dual-porosity medium and matrix, K_{fi} is the permeability of fractures when the fracture and fluid seepage direction is parallel, θ is angle between the fracture and fluid flow direction. Schlumberger Company [9] used the formation micro-scanner log (FMI) data and the fracture characterization method from Chen [9] and proposed permeability mode: $K = (aV^2/A)\varepsilon_t + b\varepsilon_t^2$, where K is the well logging permeability, where V and A are respectively the volume and area of fractures in cores per unit volume, a and b are the constants, and ε_t is porosity of the dual-porosity medium. However, these models are not quantitatively related to micro-parameters of a dual-porosity medium, such as pore size, fractures density, the length and aperture of fractures. Therefore, the seepage mechanisms in the media are not well understood.

Actually, fracture distribution, aperture, orientation and length are, in general, random in nature. For characterization of disorder and irregular objects, fractal geometry and technique are believed to be effective in science and engineering, for instance, thermal conductivity [10–12], permeability [13,14] and spontaneous imbibition [15,16]. It has been shown that porous media can be

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described by fractal geometry theory. The pores [17,18] in porous media and fractures [19,20] have been shown to be fractals. Yu et al. [21] investigated the seepage characteristics in porous media based on fractal geometry theory for porous media and Hagen–Poiseuille flow in tortuous capillaries, and they obtained the analytical expression for permeabilities of particle-like porous media and porous fabrics. Their fractal model can reveal more mechanisms of fluid flow in porous media than traditional model. Yun et al. [22] studied non-Newtonian fluid flow characteristics in fractal porous media, including starting pressure gradient and permeability etc. Xiao et al. [23] studied gas–water relative permeability of fibrous gas diffusion layer (GDL) with a very high porosity (i.e. from 0.7 to 0.95) in proton exchange membrane fuel cells (PEMFC) by using the fractal geometry. Hao and Cheng [24] simulated the permeabilities in carbon paper gas diffusion layers by applying the Lattice Boltzmann methods and the fractal theory for porous media. Cai et al. [25] derived analytical model for spontaneous imbibition in fractal porous media including gravity. The effect of capillary pressure is very important in unsaturated porous media. Miao et al. [26] investigated spherical seepage in unsaturated fractal porous media with capillary pressure included. However, the above models mentioned are not involved in fracture networks.

Xu et al. [27,28] obtained the analytical expressions for permeability and thermal conductivity of fractal-like tree networks. Wang and Yu [29], Zheng and Yu [30] established special dual-porosity medium models respectively for the starting pressure gradient for Bingham fluids in porous media and for gas flow through dual-porosity media. However, the fractal-like tree network in their model is assumed to be a kind of ideal and symmetrical network, whereas the actually fracture networks are random and irregular.

Watanbe and Takahashi [31] studied the effect of microscopic parameters of fractal fracture networks on permeability and heat extraction in hot dry rocks. However, they did not provide an expression for permeability of fracture networks. Jafari and Babadagli [32] used multiple regression analysis and obtained the permeability expression (with several empirical constants) for random fractures by fractal geometry based on the well logging and observed data. However, their expression does not include the tortuous factor and microstructure parameters of fracture networks.

Recently, Miao et al. [33] used fractal geometry theory and technique to study seepage characteristics and got the analytical expression for permeability of fracture networks. The model includes microstructure parameters of fractures and does not contain any empirical constant. Most recently, Liu et al. [34] studied the permeability of fractal fracture networks with Monte Carlo method. However, above models are for the fracture networks without including matrix permeability.

In order to well understand the seepage mechanisms in dual-porosity porous media, in this work, an analytical model is proposed based on the fractal geometry theory and technique for porous media with the length distribution of fractures obeying the fractal scaling law. This work focuses on seepage characters in saturated dual-porosity media, in which a liquid (e.g. water) is fully filled and the capillary pressure can be thus neglected. In addition, in this paper, the porous matrix is assumed to consist of a bundle of tortuous capillaries whose size distribution follows the fractal scaling law, while the fractures in lengths are random and follow the fractal scaling law. In Section 2, we introduce the basic fractal theory for porous media. Then, Characterization of fractures is described in Section 3. The permeability model for the Newtonian fluid flow in the media is derived in Section 4 based on the fractal geometry theory for porous media and basic seepage law (cubic law). In Section 5, the validity of the proposed model is verified by comparison between the model predictions and experimental data, and the parametric study is also shown in this section. The conclusions from this work are given in Section 6.

2. Fractal theory for porous matrix

In generally, the microstructures and tortuous flow pathways in porous media are randomly distributed and extremely complicated. It is, therefore, very difficult to analytically study the seepage characteristics in porous media. Fortunately, the micro-pores in porous media have fractal characters [17,18] and the fractal geometry for porous media has received the great success in analysis of flow and transport properties in porous media [21–26].

It has been shown that the cumulative number of pres/capillaries whose diameters are greater than or equal to λ is given by the fractal scaling law [35,36]:

$$N(L \geqslant \lambda) = (\lambda_{\max}/\lambda)^{D_f} \tag{1}$$

where $\lambda_{\max}$ is the maximum capillary/pore diameter, D_f is the fractal dimension for the size distribution of pores/capillaries.

Since there usually are large numbers of capillaries or pores in porous media, Eq. (1) can be regarded as continuous and differentiable functions. So, differentiating Eq. (1) with respect to λ results in the number of capillaries/pores whose diameters are in the interval of λ and $\lambda + d\lambda$:

$$-dN = D_f \lambda_{\max}^{D_f} \lambda^{-(D_f+1)} d\lambda \tag{2}$$

where $-dN > 0$. Equation (2) indicates that the number of capillaries/pores decreases with the increase of capillary/pore sizes.

The porosity and the fractal dimension are related by [35,36]

$$\varepsilon_m = (\lambda_{\min}/\lambda_{\max})^{d_E - D_f} \tag{3}$$

where ε_m is the effective porosity of a porous matrix, $\lambda_{\min}$ is the minimum capillary/pore diameter, d_E is the Euclid dimension, and $d_E = 2$, $0 < D_f < 2$ in two dimensions, and $d_E = 3$, $0 < D_f < 3$ in three dimensions.

Capillaries in porous media are usually tortuous, and the diameter and length of tortuous capillaries follow the fractal scaling law [35,36]:

$$L_t(\lambda) = \lambda^{1-D_T} L_0^{D_T} \tag{4}$$

where $L_t(\lambda)$ and L_0 are respectively the actual length and straight length of a capillary. D_T is the fractal dimension for tortuosity of tortuous capillaries. $D_T = 1$ represents a straight capillary. $1 < D_T < 2$ refers in two dimensions and $1 < D_T < 3$ in three dimensions. A higher value of D_T corresponds to a highly tortuous capillary. The fractal dimension for tortuosity can be expressed as [36,37]

$$D_T = 1 + \frac{\ln \tau_{av}}{\ln(L_0/\lambda_{av})} \tag{5}$$

where λ_{av} is the average diameter of capillaries/pores in porous media, τ_{av} is the average tortuosity of capillaries. Tortuosity of the tortuous capillary is often defined by [38]

$$\tau = L_t/L_0 \tag{6}$$

The average diameter of capillaries/pores can be determined by [22]:

$$\lambda_{av} = \lambda_{\max}\left(\frac{D_f}{4 - D_f}\right)^{1/4} \tag{7}$$

The average tortuosity of capillaries can be calculated by [39]

$$\tau_{av} = \frac{1}{2}\left[1 + \frac{1}{2}\sqrt{1-\varepsilon_m} + \sqrt{1-\varepsilon_m}\,\frac{\sqrt{\left(\frac{1}{\sqrt{1-\varepsilon_m}} - 1\right)^2 + \frac{1}{4}}}{1 - \sqrt{1-\varepsilon_m}}\right] \tag{8}$$

The total pore area is obtained by