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A fractal analysis of laminar flow resistance in roughened microchannels

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ABSTRACT

The fractal geometry theory and method are used to simulate the rough surface topography. The pressure gradients, friction factors and Poiseuille numbers for laminar flow through microchannels with roughened surfaces are derived. The proposed fractal models for the relative increases of the pressure gradient, friction factor, and the Poiseuille numbers of laminar flow through microchannels with roughened surfaces are found to be a function of the microstructural parameters of roughness surfaces. These parameters are the fractal dimension, maximum peak, minimum peak, and the ratio of the minimum diameter to the maximum diameter of conic peaks on roughened surfaces. Moreover, every parameter in the proposed models has clear physical meaning. The model predictions are compared with those of the existing measurements. Fair agreement between the fractal model predictions and experimental data is found.

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1. Introduction

In the past decades, tremendous attention has been given to fluid flow in microchannels due to the rapid development of Micro-Electro-Mechanical Systems (MEMS) such as micro-motors, micro-pumps, micro-sensors, micro-valves, micro-turbines and micro-actuators. Flow characteristics (velocity distribution and pressure drop) have the remarkable effect on the design and process control of MEMS [1–3] and heat transfer processes [4,5].

Pfahler et al. [6,7] carried out the experimental investigation on fluid flow in microchannels, and they concluded that for larger channels the experimental data showed the excellent agreement with the classical theory, however, as the channel diameter is reduced, an increasing deviation from the classical theory was observed. Measurements [1,8–13] have shown that the Poiseuille number (or the friction constant $f \cdot \mathrm{Re}$) of laminar flow in rectangular microchannels are higher than that of the classical theory [14], and the same conclusion was obtained by other researchers [15–18] in cylindrical microchannels. This phenomenon may be attributed to surface roughness effects. Mala and Li [8] studied water flow through microtube of fused silica (FS) and stainless steel (SS), and they proposed a roughness-viscosity model to interpret the experimental phenomena. However, this model contains as many as 4 empirical/fitting constants. Qu and Mala [12] also carried out a similar study and introduced the roughness viscosity in the equation of motion. However, the roughness viscosity obtained by fitting their experimental data has as many as 5 empirical/fitting constants.

It has been shown that the rough surfaces have the fractal characters and can be described by fractal geometry and technique [19–22]. Majumdar and Bhushan [20] proposed the size distribution of contact spots on engineering surface by fractal geometry theory. Warren and Krajcinovic [21] applied the random Cantor set to simulate the elastic-perfectly plastic contact of rough surfaces. He and Zhu [22] characterized the rough surfaces by W–M function [20] and by using the fractal dimension and characteristic length. Chen and Cheng [23] measured the fractal dimension D for roughness surfaces profiles in microchannels used by Pfund et al. [10], and they found that the Poiseuille number in roughened microchannels is a function of the classical Poiseuille number, average height of rough elements and hydraulic radius. But, the proposed expression has two empirical constants determined by fitting Pfund et al.'s experimental data. Bahrami et al. [3] proposed that the wall roughness posses a Gaussian isotropic distribution. Zhang et al. [24] and Chen et al. [25] applied the Weierstrass–Mandelbrelbrot function to characterize the multiscale self-affine roughness. Recently, Chen et al. [26] described the topography of the rough surface by using a Cantor set to numerically simulate heat transfer in the microchannels. Chen et al. [27] introduced the fractal W–M function to characterize the multiscale self-affine rough surface of microchannels and evaluate the role of the rough surface structure on the thermal and hydrodynamic properties by using a computational fluid dynamic simulation. Zhang et al. [28] used a lattice Boltzmann simulation of gas slip flow incorporating rough surface effects with a focus on gas–solid interaction.

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Nomenclature

D	the fractal dimension for the base diameter distribution of cone-shaped peaks/spots.
N	the number of conic peak
V_i	the volume of a cone-shaped peak/spot
V_t	the total volume in a set of fractal cones
S_1	the total base area of cones for a set of fractal cones
S_0	the total area for a unit cell
S_i	the base area for a cone-shaped peak/spot
L_0	the side length of a unit cell
R	the radius of the cylindrical microchannel
b	the height in a smooth rectangular microchannel
W	the width in a smooth rectangular microchannel
$\bar{h}_{eff}$	the effective average height of cones/roughness elements
h	the height of conic peak
u_m	the mean velocity over the cross section in a smooth microchannel
Q	the volume flow
τ_w	the wall shear
f	friction factor
Re	the Reynolds number
D_h	hydraulic diameter
Po	Poiseuille number
λ	the base diameter of conic peak
λ_{min}	the minimum diameter of conic peak
λ_{max}	the maximum diameter of conic peak
ξ	the ratio of height to base diameter of conic peak
α	the ratio of the minimum diameter to the maximum diameter
ϕ_s	the area ratio of the total base area of all cone-shaped peaks/spots to the whole surface area of a unit cell the dynamic viscosity
ε_r	the *relative roughness* in rectangular roughened microchannels
ε_c	the *relative roughness* in cylindrical roughened microchannels
L	representative length
β	the relative increase of the pressure gradient

Subscripts

c	cylindrical
cs	cylindrical smooth microchannels
R	rectangular roughened microchannels
cR	cylindrical roughened microchannels

In this work, a fractal model for flow friction in microchannels is derived based on the assumption that the roughened surfaces in microchannels have the statistically isotropic and self-similar fractal characters. The theoretical predictions of flow resistance from the proposed model will be compared with the existing experimental data in microchannels with roughened surfaces.

2. Fractal theory for roughened surfaces

Mandelbrot [29] in his famous book "The Fractal Geometry of Nature" proposed that the cumulative size distribution of islands on earth's surface follows the fractal scaling law: $N(A > a) \sim a^{-D/2}$, where N is the total number of islands of area A greater than a, and D is the fractal dimension for size distribution of islands on Earth's surface. Marjumdar and Bhushan [20,30] used the fractal scaling law to describe the contact spots (roughness elements) on engineering surfaces, and the fractal scaling law is

$$N(L \geqslant a) = (a_{max}/a)^{D/2} \tag{1}$$

where a and a_{max} are respectively the spot area and maximum spot area on engineering surfaces, L is the scale of measurement, and D is the fractal dimension for area distribution of spots, and $0 < D < 2$. If $D = 2$, the spots cover a two-dimensional plane. If $D = 0$, this refers to a smooth surface because the number of spot on a surface is approximately zero, compared to that as $D = 2$. Yu and Cheng [31] extended Eq. (1) for describing the spots on engineering surfaces to describe the pores in porous media.

Warren and Krajcinovic [21] and Chen et al. [26,32] described the self-affine topography of the rough surface by using a Cantor set. As we know, self-affine has unequal scaling in different directions, and Manelbrot [29] showed that the dimension of self-affine curves can be obtained from their power spectra; Marjumdar and Bhushan [30] explained the difference between the self-similarity and self-affinity. They wrote: "The definition of self-similarity is based on the property of *equal* magnification in all directions". That means the fractal dimension is the same in all directions for self-similarity fractals. However, for self-affine fractals, they pointed out that "there are many objects in nature which have unequal scaling in different directions". This implies that the fractal dimension is different in different directions.

Some investigators [33,34] showed that rough elements (conic peaks) on rough surfaces are statistically self-similar fractals. Therefore, in this work, we also assume that peaks on a surface do not overlap each other and are statistically self-similar fractals, not self-affine fractals, which will be our next work and will be addressed elsewhere. In this work, we assume that the spots on surfaces are cone-shaped as shown in Fig. 1 [35], and the ratio of height to base diameter of conic peak is $\xi = h/\lambda$.

Since this work assumes that rough elements (conic peaks) on rough surfaces have the statistically self-similar fractal characteristic, the base diameter of cone-shaped distribution also follows the fractal scaling law, and then Eq. (1) can be modified as

$$N(d \geqslant \lambda) = \left(\frac{\lambda_{max}}{\lambda}\right)^D \tag{2}$$

where λ and λ_{max} are respectively the base diameter and maximum base diameter, D is the fractal dimension for the base diameter distribution of cone-shaped peaks/spots. In Eq. (2), $0 < D < 2$, and $D = 2$ means that a surface is so rough (or a surface is covered so many rough elements) that the surface profile makes a channel's diameter become as possibly as small, i.e. the actual cross-sectional area for flow in the channel becomes as possibly as small, and $D = 0$ corresponds to a smooth surface.

In general, there are numerous elements of roughness on roughened surfaces. Eq. (2) can thus be considered as continuous and differentiable equation. By differentiating with respect to λ, the number of roughness elements in the infinitesimal range of λ to $\lambda + d\lambda$ can be found to be

$$-dN = D\lambda_{max}^{D}\lambda^{-(D+1)}d\lambda \tag{3}$$

where $-dN > 0$. Eq. (3) indicates that the number of roughness elements increases with the decrease of sizes.

From Eq. (2), the total number of roughness elements from the minimum diameter λ_{min} to the maximum diameter λ_{max} can be obtained by

$$N_t(\lambda \geqslant \lambda_{min}) = \left(\frac{\lambda_{max}}{\lambda_{min}}\right)^D \tag{4}$$

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