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Mass transfer from two spheres in tandem accompanied by chemical reactions

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HIGHLIGHTS

- Mass transfer with chemical reactions from two spheres in tandem to a fluid flow.
- Conditions when the interaction between spheres does not affect the mass transfer.
- The mass transfer rate depends on the Henry number, *H*.
- For finite values of *H*, mass transfer depends on the order of the chemical reaction.

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ABSTRACT

The mass transfer from two spheres in tandem to an incompressible fluid flow accompanied by chemical reactions (first-order and second-order irreversible) into the fluid was analysed numerically. Creeping flow was assumed. The spheres have the same diameter, initial concentrations and physical properties. The concentration of the transferring species inside the spheres is considered: (1) constant and (2) spatially uniform and varying in time. The mathematical model equations were solved numerically in bispherical coordinates system. The computations focused on the conditions when the interaction between spheres does not influence the mass transfer rate.

1. Introduction

In many industrial, life and environmental processes, mass transfer in two – phase flow often occurs accompanied by a chemical reaction. The control of this process requires a comprehensive understanding of the transport phenomena between the two phases close to the interface. The comprehensive understanding of these phenomena requires the accurate prediction of interfacial transfer rate due to simultaneous diffusion, convection and chemical reaction.

The mass transfer accompanied by chemical reactions between a sphere and a surrounding fluid flow is the topic of a large number of articles published over the years. For this reason, the present literature review is restricted to articles that analyse cases similar to that investigated in the present work, i.e. articles that assume: (1) the transfer resistance inside the sphere is negligible (the concentration of the transferring species inside the sphere is spatially uniform/constant); (2) the chemical reaction takes place only in the surrounding fluid.

Johnson and Akehata (1965) were the first to study the mass transfer accompanied by a first – order irreversible chemical reaction from a sphere in creeping flow. Their analysis was extended to larger Reynolds numbers and second order chemical reaction by Johnson et al. (1967). Ishii and Johnson (1969) analysed the wall effect on the mass transfer accompanied by a first – order irreversible chemical reaction from a sphere in potential flow. Shah (1972) analysed the case of a complex chemical reaction for a bubble in creeping flow using the boundary layer formalism. Numerical solutions for different types of chemical reactions can be viewed in Gartsman et al. (1979); Klinzing et al. (1980); Juncu and Mihail (1989); Khinast (2001); Khinast et al. (2003); Wylock et al. (2011); Pigeonneau et al. (2014); Grunding et al. (2016). In Khinast et al. (2003); Wylock et al. (2011); Pigeonneau et al. (2014) the deformation of the bubble was taken into consideration.

The dissolution of a stationary gas bubble in the presence of a chemical reaction was investigated in Subramanian and Chi (1980); Weinberg and Subramanian (1981); Yoshikawa et al. (1998). Subramanian and Chi (1980) have shown that the agreement between the full model and approximate models that consider the radius of the bubble constant is very good for moderate

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Nomenclature

A	dimensionless concentration of the reactant transferring from the spheres in the surrounding fluid, m_A/m_{A0}
B	dimensionless concentration of the reactant soluble only in the surrounding fluid, $m_B/m_{B\infty}$
c	characteristic length, bi-spherical coordinate system, (m)
$\bar{c}$-	dimensionless characteristic length, bi-spherical coordinate system, $2c/d$
d	sphere diameter, (m)
D	Fickean self-diffusion coefficient ($m^2\ s^{-1}$)
Da	Damkholer number, $Da = k\ d^2/4\ D_A$ for reaction (1a), $Da = k\ d^2\ m_{A0}/4\ D_A$ for reaction (1b), $Da = k\ d^2\ m_{B0}/4\ D_A$ for reaction (1c), dimensionless
E	enhancement factor, $E = Sh\ (Da \neq 0)/Sh\ (Da = 0)$, dimensionless
H	Henry number for species A, dimensionless
k	chemical reaction rate constant, (s^{-1}) for reaction (1a) or ($m^3\ kg^{-1}\ s^{-1}$) for reactions (1b, c)
L	distance from the center of the sphere to the origin of the bi-spherical coordinate system, (m)
M	mass concentration ($kg\ m^{-3}$)
Pe	Peclet number, $Pe = U_\infty\ d/D_A$, dimensionless
q	dimensionless group, $q = m_{A0}/m_{B\infty}$
r	reaction rate, ($kg\ m^{-3}\ s^{-1}$)
R_{BA}	diffusivity ratio, D_B/D_A, dimensionless
Sh	Sherwood number, dimensionless
t	time, (s)
U_∞	free stream velocity, ($m\ s^{-1}$)

Greek Letters

δ	dimensionless distance from the center of the sphere to the origin of the bi-spherical coordinate system, $\delta = 2\ L/d$
η	coordinate in bi-spherical coordinate system
λ	reduction factor, Sh_i/Sh^{iso}, $i = 1, 2$, dimensionless
ξ	coordinate in bi-spherical coordinate system, (rad)
τ	dimensionless time or Fourier number, $\tau = 4t\ D_A/d^2$
ψ	dimensionless stream function

Subscripts

A	refers to reactant A
B	refers to reactant B
crt	refers to critical value
s	refers to the surface of the spheres
τ	refers to time average quantity
0	initial conditions
1	upstream (leading) sphere
2	downstream (trailing) sphere
∞	large distance from the spheres

Superscripts

iso	refers to an isolated, single sphere

to large values of the dimensionless reaction rate parameter. Several approximate solutions used to describe the time variation of the bubble radius have been discussed by Weinberg and Subramanian (1981).

In references (Johnson and Akehata, 1965; Johnson et al., 1967; Ishii and Johnson, 1969; Shah, 1972; Gartsman et al., 1979; Klinzing et al., 1980; Juncu and Mihail, 1989; Khinast, 2001; Khinast et al., 2003; Wylock et al., 2011; Pigeonneau et al., 2014; Grunding et al., 2016; Subramanian and Chi, 1980; Weinberg and Subramanian, 1981; Yoshikawa et al., 1998) the concentration of the transferring species inside the sphere was considered constant. The mass transfer accompanied by a chemical reaction from a sphere with spatially uniform but varying in time concentration was analysed in Ruckenstein et al. (1971); Soung and Sears (1975); Juncu (2001); Juncu (2012). Ruckenstein et al. (1971) derived analytical expressions for the Sh number and the sphere average concentration using the boundary layer approximation. The expressions for the Sh number were deduced assuming constant concentration inside the sphere. Soung and Sears (1975) solved numerically the same problem for different orders of chemical reaction and neglecting the mass transfer by diffusion in the tangential direction. Numerical solutions for the mass transfer from a fluid sphere in creeping flow with spatially uniform but varying in time concentration accompanied by an isothermal, first – order, irreversible chemical reaction in the surrounding fluid phase can be viewed in Juncu (2001, 2012).

In almost all practical applications, the mass transfer takes place in sphere swarms. For sphere swarms, the single sphere data cannot predict accurately the mass transfer rate for all situations Koynov et al. (2005); Radl et al. (2008). In this case, the effect of the interaction between spheres on the mass transfer rate should be analysed. A starting point for this analysis is the study of the mass transfer for the classical, standard, two spheres interactions, i.e. tandem (or in – line), transverse (side – by – side) and staggered, (Juncu, 2007).

Mass transfer accompanied by a chemical reaction from two spheres which are moving in close proximity was analysed until now only for the case of surface reaction (carrier – extraction) and tandem interaction in creeping flow (Juncu, 2010). The aim of this work is to make a new step in this direction considering the mass transfer from two spheres in tandem accompanied by irreversible chemical reactions (first order and second order) in the surrounding fluid flow. The concentration of the transferring species inside the sphere is considered constant or spatially uniform but varying in time. The influence of Damkohler number on the mass transfer rate is investigated for creeping flow, different spheres' spacing and $Pe = 10, 100, 1000$. From our knowledge, this problem was not investigated until now.

2. Model equations

Consider the steady, axisymmetric, creeping flow of a Newtonian incompressible fluid past two spheres of equal and constant diameter d, parallel to their line of centers. Oscillations and rotation of the spheres do not occur during the movement. The interface between the spheres and the fluid is assumed rigid. The physical properties of the spheres and the ambient medium are uniform, isotropic and constant. Also, the effects of free convection, Marangoni convection and cross – diffusion are negligible.

From the spheres, the chemical species A transfers into the surrounding fluid. The initial concentration of chemical species A inside the spheres is the same. Also, during the mass transfer process, the concentration of A inside the spheres is spatially uniform. In the surrounding fluid, one of the following chemical reactions

$$A \rightarrow products, \quad r = km_A \tag{1a}$$

$$A \rightarrow products, \quad r = km_{A^2} \tag{1b}$$

$$A + B \rightarrow products, \quad r = km_A m_B \tag{1c}$$