Chemical Engineering Science 187 (2018) 27–32

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Theoretical analysis of the viscosity correction factor for heat transfer in pipe flow

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HIGHLIGHTS

- An updated form of the Sieder-Tate viscosity correction factor is obtained.
- Nusselt number relationships for the hot and cold wall are distinct.
- A general and exact solution for any range viscosity ratios is presented.
- Asymptotic forms closely relate to the exact solution for moderate viscosity ratios.

ARTICLE INFO

Article history:
Received 8 November 2017
Received in revised form 20 April 2018
Accepted 21 April 2018
Available online 25 April 2018

Keywords:
Sieder-Tate
Heat transfer
Boundary layer
Viscosity correction factor
Nusselt number

ABSTRACT

In most heat transfer applications, knowledge of the viscosity variation is important. Thus viscosity correction factors have been researched and proposed for almost a century. One of the most successful relations was reported by Sieder-Tate in 1936, which has been widely used in engineering analysis and design. In this study, we have improved on the Sieder-Tate relation, following a classical theoretical analysis of the thermal boundary layer. An exact solution to the viscosity correction factor was obtained which shows that the Sieder-Tate correction factor over-predicts the heat transfer coefficient in the case of cold wall (cooling) and does not hold properly for hot wall (heating). We have found that a relation of $\left(\frac{\mu_\infty}{\mu_w}\right)^{0.254}$ (in case of cooling) and $\left(\frac{\mu_\infty}{\mu_w}\right)^{0.087}$ in the case of heating is better than the Sieder-Tate factor of $\left(\frac{\mu_\infty}{\mu_w}\right)^{0.14}$ (uniformly applied to both the heating and cooling cases). Here μ_∞ and μ_w are the bulk and wall viscosity coefficients respectively. The theoretical analysis also shows that the above correction factors are limited to small values of $\ln\left(\frac{\mu_w}{\mu_\infty}\right)$ (for cold wall) and $\ln\left(\frac{\mu_\infty}{\mu_w}\right)$ (for hot wall). However a general solution has been obtained and the correlation developed by Petukhov (1970) closely matches the exact solution for the case of cold wall cooling of a fluid.

1. Introduction

Convective heat transfer (heating or cooling) of fluids in a pipe flow is inevitably present in any processing industry, for example in heat exchangers, coolant flows, power generators, etc. However, design of such systems or any heat transfer equipment requires, amongst other inputs, knowledge of heat transfer coefficients which depend on the properties of the fluid, flow characteristics and geometry. The viscosity of the fluid depends strongly on temperature, and so the thermal gradient across the boundary layer creates a spatial variation in viscosity. In the case of cooling, the dynamic viscosity at the wall (μ_w) is larger than the bulk viscosity (μ_∞), whereas when the fluid is heated, $\frac{\mu_w}{\mu_\infty} < 1$. One of the early correlations reported by Sieder and Tate (1936) includes what is probably the most widely used viscosity correction factor in engineering heat transfer design. They suggested that $\left(\frac{\mu_\infty}{\mu_w}\right)^{0.14}$ should be the viscosity correction factor to be applied to the Nusselt number (Nu) for both heating and cooling situations. This was obtained by correlating heat transfer data with the mainstream fluid properties and temperature. In the work by Petukhov (1970), the viscosity correction factor obtained empirically by non-linear fitting to experimental data generated two exponents: 0.11 in the case of heating, and 0.25 in the case of cooling.

The later work by Field (1990) presents viscosity corrections obtained theoretically from classical boundary layer analysis. Field found that the correction factors, for small or moderate values of

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https://doi.org/10.1016/j.ces.2018.04.047
0009-2509/

Nomenclature

c_p	specific heat transfer capacity of the fluid, $J \cdot kg \cdot K$
d_e	hydraulic diameter of the pipe, m
F	correction factor, as defined in Eq. (12)
h	heat transfer coefficient, W/m^2 K
$\bar{h}$	spatially averaged heat transfer coefficient, W/m^2 K
k	thermal conductivity of the boundary wall, $W/m \cdot K$
Nu	Nusselt number, defined as $\frac{\bar{h}d_e}{k}$
p	exponent for the viscosity relation μ_∞/μ_w
Pr	Prandtl number, defined as $c_p\mu_\infty/k$
q_0	heat flux across the wall, W/m^2
Re	Reynolds number, defined as $\frac{\rho u_m d_e}{\mu_\infty}$
T	temperature of fluid, K
T_∞	temperature in the bulk fluid, K
T_w	temperature of the boundary wall, K
u	fluid velocity, m/s
$\tilde{u}$	non-dimensional fluid velocity, equals $u\frac{\mu_w}{\tau_w\delta_T}$
u_1	maximum fluid velocity, m/s
u_m	mean fluid velocity in the channel, m/s
x	Abscissa of the coordinate, m
y	co-ordinate from the wall, m
$\tilde{y}$	scaled y-coordinate with respect to δ_T

Greek symbols

α	logarithm of the viscosity ratio, equals $\ln(\mu_w/\mu_\infty)$
δ	hydrodynamic boundary layer thickness, m
δ_T	thermal boundary layer thickness, m
θ	non-dimensional temperature, equals $\frac{T_w-T}{T_w-T_\infty}$
μ	viscosity of the fluid, Pa·s
μ_∞	viscosity of the bulk fluid, Pa·s
μ_w	viscosity of the fluid at the wall ($y = 0$), Pa·s
ρ	density of the fluid, kg/m^3
τ	shear stress of the fluid, Pa
τ_w	wall shear stress, Pa
ψ_0	non-dimensional parameter, equals $\int_0^1 \tilde{u}(1-\theta)d\tilde{y}$

$\frac{\mu_\infty}{\mu_w}$, are in the form of $\left(\frac{\mu_\infty}{\mu_w}\right)^p$; in the case of cooling, $p = 0.1$ whereas for heating, $p = 0.27$. These correction factors were considered superior to those obtained by Sieder and Tate (1936) and Petukhov (1970), and the influence of viscosity variations on condensation was explored by Field (1992). However, in deriving the correction factors, two inherent assumptions were made with respect to the thermal boundary layer. In following the classical heat transfer analysis they were: (i) the spatial variation of the temperature is linear in space and (ii) the shear stress is assumed to be constant. Improving on these two assumptions, was the motivation of the present work.

For a Newtonian fluid the theoretical analysis by Yang (1962) generated an index of 0.11. As noted by Joshi (1978) there is a substantial discrepancy between this result and that of Shannon and Depew (1969) who performed a similar analysis for the case of a uniform heat flux boundary condition; their results gave an index of 0.3 at the entrance to the tube decreasing to 0.14 in the fully developed region. Joshi (1978) concluded that further work was needed to clarify the situation but the focus of his own work was on non-Newtonian fluids. That Yang (1962) and Shannon and Depew (1969) had significantly different results may well be due to the former considering only heated tubes. The present work clearly considers the separate cases of a heated wall and a cold wall. In the present work, we have assumed that the thermal boundary layer is smaller than the hydrodynamic boundary layer, $Pr > 1$, which is true for most liquids (other than liquid metals) and concentrated upon Newtonian fluid in the laminar regime. We have considered here a fully developed hydrodynamic profile and a developing thermal boundary layer. Furthermore, for many liquids such as oils, the thermal boundary layer will be thin compared to the thickness of the hydrodynamic boundary layer because the ratio (δ_T/δ) depends upon $\mathrm{Pr}^{-1/2}$. As a result much of the thermal boundary layer will be within the laminar sub-layer; we return to this point later.

2. Theoretical analysis

In classical thermal boundary layer analysis for the estimation of the heat transfer coefficient, a quadratic temperature profile is considered, as given by Schlichting and Gersten (2003), Hartnett et al. (1998), Kay and Nedderman (1985)

$$\theta = 2\tilde{y} - \tilde{y}^2 \tag{1}$$

where $\theta = \frac{T_w-T}{T_w-T_\infty}$ is the scaled temperature, T_w is the boundary wall temperature and T_∞ is the bulk temperature. Here $\tilde{y} = y/\delta_T$ is the scaled y-coordinate from the wall with respect to the thermal boundary layer thickness. In extending the classical heat transfer analysis of Kay and Nedderman (1985) which did not allow for spatial viscosity variations, Field (1990) made two assumptions:

(i) to account for the linear temperature dependence on the viscosity of the fluid in the thermal layer ($\tilde{y} < 1$), the viscosity is considered dependent on the spatial variable ($\tilde{y}$) as

$$\frac{\mu}{\mu_w} = \exp(-\alpha\tilde{y}), \tag{2}$$

where μ_w is the viscosity of the fluid at the wall ($\tilde{y} = 0$) and α is a function of the ratio of the change of the viscosity between the bulk and the wall such that $\alpha = \ln(\mu_w/\mu_\infty)$. When the wall cools the fluid, $\alpha > 0$; and when the fluid is heated $\alpha < 0$.

(ii) The shear stress, τ is considered constant within the thermal boundary layer, $0 < \tilde{y} < 1$, which is reasonable if the velocity is linear within the thermal boundary layer, and holds satisfactorily for large Prandtl number.

However, for $Pr > 1$, which is true for most liquid flows (Lienhard and Lienhard, 2017), we relax both of the above assumptions. We define the viscosity dependence on temperature of the fluid as

$$\frac{\mu}{\mu_w} = \exp(-\alpha\theta) = \exp[-\alpha(2\tilde{y} - \tilde{y}^2)], \tag{3}$$

and considering an isothermal flow of a Newtonian fluid, the shear stress linearly decreases with distance from the wall

$$\tau = \tau_w(1 - \tilde{y}), \tag{4}$$

where τ_w is the wall shear stress. Using the stress-strain constitutive relationship of a Newtonian fluid $\tau = \mu\frac{du}{dy}$, together with Eqs. (3) and (4), we can write

$$(1 - \tilde{y}) = \exp(-\alpha\theta)\frac{d\tilde{u}}{d\tilde{y}}, \tag{5}$$