Chemical Engineering Science 134 (2015) 1–11

Contents lists available at ScienceDirect

Chemical Engineering Science

journal homepage: www.elsevier.com/locate/ces

Absolute and convective instabilities in double-diffusive two-fluid flow in a slippery channel

Sukhendu Ghosh [a], R. Usha [a,*], Kirti Chandra Sahu [b]

[a] Department of Mathematics, Indian Institute of Technology Madras, Chennai 600036, India
[b] Department of Chemical Engineering, Indian Institute of Technology Hyderabad, Yeddumailaram 502 205, Telangana, India

HIGHLIGHTS

- Spatio-temporal instability of two-fluid flow in a slippery channel is investigated.
- Absolutely unstable mode is found when a highly viscous fluid is placed near the wall.
- The instability can be either enhanced or suppressed by wall slip.

ARTICLE INFO

Article history:
Received 4 February 2015
Received in revised form
20 April 2015
Accepted 25 April 2015
Available online 2 May 2015

Keywords:
Laminar flow
Mixing
Stability
Fluid mechanics
Hydrodynamics
Multiphase flow

ABSTRACT

Spatio-temporal instability of miscible two-fluid symmetric flow in a horizontal slippery channel is considered. Both fluids have the same density but different viscosity. A smooth viscosity stratification is created by a thin mixed layer between the fluids due to the presence of two species/scalars, which are diffusing at different rates. Our study suggests the existence of a rapidly growing absolute unstable mode for higher viscosity ratio with a highly viscous fluid close to the slippery channel wall. This instability is less stronger in the case of the equivalent single component two-fluid flow. The viscosity stratified single component (SC) and double-diffusive (DD) slippery flows are absolutely unstable for a wide range of parameter values, when a highly viscous fluid is adjacent to the slippery wall and the mixed layer is close to the channel wall with slip. The instability can be either enhanced or suppressed by wall slip and this is dependent on the location of mixed layer, inertial effects, diffusivity and the log-mobility ratios of the faster and slower diffusing species. This suggests that one can achieve early transition to turbulence due to the absolute instability in a viscosity stratified channel flow by making the channel walls hydrophobic/rough/porous with small permeability, which can be modelled by the Navier-slip condition.

1. Introduction

The linear stability characteristics of a double-diffusive two-fluid three-layer channel flow (the equivalent core-annular configuration in the case of a pipe) with velocity slip at the walls of the channel have been recently investigated by Ghosh et al. (2014b). The flow system has two miscible fluids with two species having different diffusivity coefficients. The inhomogeneities in solute concentration have been accounted for in terms of viscosity stratification (Turner, 1974; Huppert, 1971; May and Kelley, 1997; Worster, 2004). In the presence of double-diffusive (DD) effects, the flow becomes unstable at low Reynolds numbers for a wide range of wave numbers, when the less viscous fluid is adjacent to the slippery channel wall. This is in striking contrast to the stabilization of the flow that is observed for configurations where the viscosity stratification is due to the presence of a single species or scalar (Ghosh et al., 2014a) (referred as single-component (SC) system). Further, at Reynolds numbers smaller than the critical Reynolds number for the classical Tollmien–Schlichting (TS) mode, a new unstable mode (namely DD mode) has been shown to exist, as the mixed layer of fluids moves towards the channel wall. The DD mode is dominant when the mixed layer overlaps with the critical layer (the location where the phase speed of the most unstable disturbance equals the mean velocity). Depending on the flow parameters, the velocity slip at the channel wall has stabilizing or destabilizing influence on the DD flow system. This demonstrates an effective way to control three-layer miscible two-fluid flow in a slippery channel with viscosity stratification.

If disturbances grow locally as well as spread in both upstream and downstream directions, the flow is considered to be an absolutely unstable flow. In this case, eventually the entire flow regime becomes unstable and the system behaves as a self-sustained

* Corresponding author.
E-mail address: ushar@iitm.ac.in (R. Usha).

http://dx.doi.org/10.1016/j.ces.2015.04.049
0009-2509/

resonator, oscillating at an intrinsic frequency. In contrast, disturbances amplify as they advect downstream, away from their initial location in a convectively unstable flow (Sahu and Govindarajan, 2012; Govindarajan and Sahu, 2014). The instabilities observed at relatively low Reynolds numbers in Ghosh et al. (2014b) for a miscible two-fluid flow system with viscosity decreasing towards the walls are convective in nature. However, the configuration with the highly viscous fluid adjacent to the slippery channel walls is unstable due to total viscosity stratification, and in this case velocity slip at the wall destabilizes the DD system (see Ghosh et al., 2014b).

This suggests that if one performs a generalized linear stability analysis in which both the temporal frequency and the spatial wave number are complex, then, it may be possible to clearly predict the boundaries that separate the convectively and absolutely unstable flows in the space of governing dimensionless parameters, such as the Reynolds number, the Schmidt number, the viscosity ratio and the location of mixed layer. In addition, knowledge of boundaries of absolutely unstable regions at relatively low Reynolds numbers may also provide information on the parameter regimes where mixing of species can be enhanced and this may be useful in relevant applications.

It is of interest, therefore, to examine whether the parameter regimes of convective and absolute instabilities predicated for DD (Sahu and Govindarajan, 2012) as well as SC (Sahu et al., 2009) flow systems in a rigid channel are influenced by the velocity slip at the channel walls. Such efforts in understanding the effects of slip at the wall in laminar channel flows gain importance due to their occurrence in many applications such as lubrication, high-speed rarefied flows (Kennard, 1938; Bird, 1994), drag reduction in microchannel flows (Tretheway and Meinhart, 2002; Choi et al., 2003; Kim and Kim, 2002), polymer melt (Denn, 2001), technological and biological drag reduction surfaces (Hoyt, 1975; Bechert et al., 2000) and microfludics (Zhu and Granick, 2001; Thompson and Troian, 1997), where the velocity of viscous fluid exhibits a tangential slip at the wall. The possibility of slip at the solid boundary has been confirmed by experimental predictions (Denn, 2001; Zhu and Granick, 2001) and molecular dynamic simulations (Thompson and Troian, 1997). In fact, the slip effects on the stability characteristics of a Poiseuille flow in a channel for a single fluid (Gersting, 1974; Spille and Rauh, 2000; Gan and Wu, 2006; Lauga and Cossu, 2005; Ling et al., 2008; Sahu et al., 2008) and for viscosity-stratified immiscible fluids (You and Zheng, 2009; Webber, 2007) have been reported and these investigations can be thought of describing the results for flow systems in channels with porous/rough/hydrophobic walls, which can be modelled by velocity slip at the surfaces (Tretheway and Meinhart, 2002; Choi et al., 2003; Min and Kim, 2005; Pascal, 1999; Neogi and Miller, 1983). In addition, the slip condition can also be modelled by eddies over wavy/rough surfaces (Wierschem et al., 2003; Scholle et al., 2004; Wierschem and Aksel, 2004; Rund et al., 2006).

It is important to understand at this stage, the corresponding results that are available for different flow configurations, which are relevant to the present study. In the case of two-dimensional Poiseuille flow of single fluid in a rigid channel, there is no absolute instability for any Reynolds number (*Re*). The flow is convectively unstable for $Re > Re_{cr}$, where Re_{cr} is the critical Reynolds number (3848.16 based on average flow rate and half channel width). These conclusions on the stability characteristics by Deissler (1987) have been based on the numerical solution of Orr–Sommerfeld system for complex frequency and complex wave number for a wide range of Reynolds numbers (*Re*) and on the asymptotic analysis for large *Re*. Valette et al. (2004) have explained the occurrence of defects at die exit, by performing a convective linear stability analysis of a two-layer coextrusion flow for molten polymers. Their analysis shows that there exists a dominant mode for which the spatial amplification rate reaches its maximum.

Linear stability analysis of miscible two-fluid channel flows reveals that such flows are unstable at low Reynolds numbers and high Schmidt numbers (Ranganathan and Govindarajan, 2001; Govindarajan, 2004). The investigation by Ern et al. (2003) for the case of continuous but rapidly varying viscosity stratification in a channel reveals the destabilizing effect of diffusion. The analysis by Sahu et al. (2009) on the convective and absolute instabilities of miscible two-fluid flow in a channel with uniform layers of highly viscous fluid adjacent to the wall and less viscous fluid in the channel core showed that the bandwidth of parameters for which the flow is absolutely unstable is increased as the diffusivity of the solute in the solvent decreases (i.e., the Schmidt number, *Sc* increases). Also, the onset of instability is at very low *Re* and the flow becomes increasingly unstable as *Sc* increases. In the above study, the viscosity stratification arises due to a pure solvent (say fluid-1) and a solution containing solute/scalar at a particular concentration (say fluid-2), which is a single component flow (SC) system.

The results for a double-diffusive case (DD) where the viscosity stratification arises due to the presence of two solutes/scalars with different diffusivities have also been examined by Sahu and Govindarajan (2011), Mishra et al. (2012), and Sahu (2013). The instability sets in at very low Reynolds number for a configuration with fluid viscosity decreasing towards the wall. These instabilities are convective in nature. Sahu and Govindarajan (2012) examined the spatio-temporal linear stability of a DD miscible two-fluid flow in a rigid channel when the viscosity increases towards the wall. The DD flow is both convectively and absolutely unstable and the growth rates are higher as compared to the corresponding SC system. Further, the regime of instability is increased in the presence of a second solute/scalar that decreases the average Schmidt number. This is in contrast to the instability that sets in at very low Reynolds number in the SC flow and which becomes unstable as the diffusivity of the solute in the solvent decreases (i.e., as the Schmidt number increases) (Sahu et al., 2009). Also the bandwidth of parameters for which the DD flow is unstable is much wider than that for SC flow. The recent review article by Govindarajan and Sahu (2014) extensively discussed instabilities in viscosity stratified flows.

The present study extends the investigation of Sahu and Govindarajan (2012) on the absolute and convective instabilities of double-diffusive miscible two-fluid flow in a rigid channel to that in a slippery channel and examines the effects of wall slip through a linear stability analysis. The analysis investigates a configuration with viscosity increasing towards the slippery channel wall. For example, if a PDMS (polydimethylsiloxane) channel is hosting a two-fluid flow of cold water and hot glycerol solution (where $R_s > 0$, $R_f < 0$; R_s, R_f are log-mobility ratios of the slower and faster diffusing scalars, namely temperature and glycerol, defined later in Section 2), then the present study addresses the spatio-temporal linear stability analysis of a DD flow system with a configuration where highly viscous fluid is near the wall (which implies $R_f + R_s > 0$). A temporal linear stability analysis by Ghosh et al. (2014b) shows that there is a large unstable region for a wide range of wave numbers due to total viscosity stratification. The present study attempts (i) to find the boundaries of convectively and absolutely unstable regions for this configuration under wall slip, and (ii) to answer the questions: are there parameter regimes where the DD system in a rigid channel is not absolutely or convectively unstable but it is so in a slippery channel and vice versa.

The analysis is also extended to the SC system. For example one can consider a PDMS (polydimethylsiloxane) channel hosting an miscible oil–water flow and perform a stability analysis for a configuration in which oil (highly viscous fluid) flows adjacent to the hydrophobic wall. For this configuration it has been shown in Ghosh et al. (2014a) that the flow system is highly unstable. In this case, surface energy will be high, the contact angle will be small (less than