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Continuity theory and settling model for spheres falling in non-Newtonian one- and two-phase media

J. Faitli

Institute of Raw Materials Processing and Environmental Process Engineering, University of Miskolc, Egyetemváros, Miskolc 3515, Hungary

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A B S T R A C T

The particle settling is a basic phenomenon: however, it determines the design of many unit operations and machines of mineral processing. A new test device has been developed in order to measure the terminal settling velocity of large steel balls settling in fine particulate solids - water mixtures. The developed inductive sensor does not influence the motion of the ball and it can be applied for non-transparent and non-Newtonian fine suspensions. A new hypothesis, namely a continuity theory for coarse disperse systems is introduced here. According to this theory, if the particles of a fine suspension are so small that they fit into the laminar sub-layer around a settling coarse particle, the fine suspension can be treated as a continuum. If they do not fit, hindered settling dominates between the coarse and fine particles. It was also recognised that if a particle settles at a constant speed in any media that is in an equilibrium state, therefore, the "equilibrium mean surficial shear stress (τ_e)" and the "equilibrium mean surficial shear rate" have been introduced. The equilibrium mean surficial shear stress can be calculated initially, because it is simply the force of gravity minus the buoyant force over three times the total surface of the particle. Once τ_e is known, the equivalent Newtonian absolute viscosity can be determined and the terminal settling velocity of particles falling in non-Newtonian media can be calculated by the known procedures for Newtonian fluids.

1. Introduction

1.1. Summary of the literature

1.1.1. Industrial importance

In mineral processing particle motion is an important fundamental phenomenon and the design of many unit operations and machines are based on the terminal settling velocity and the initial acceleration of the settling particles. One fundamental case is the particle settling in a given force field, such as gravitational, centrifugal or electrostatic. Another very important question is how a fluid flow drags particles. In chemical engineering the particles move in typically one-phase media, in liquids and gases, but these one-phase media can often be non-Newtonian because of the complex material structure. In mechanical processing, namely in mechanical waste water treatment processes, in hydraulic transport of solid granular materials and in wet mineral- and waste processing it is typical that fine particulate solids and water suspensions become non-Newtonian with increasing solids concentration (Faitli and Gombkötő, 2015).

1.1.2. Single sphere settling in a Newtonian one phase medium

The particle motion in one-phase media is widely examined in the literature (e.g. Brown and Lawler, 2003; Gumulya et al., 2007; Horsley et al., 2004; Mohammed, 2013; Song et al., 2009). The fundamental case is the undisturbed settling of a single spherical particle in a Newtonian incompressible fluid in the gravitational field. Particles typically smaller than 0.1 μm carry out molecular (Brownian) motion (colloid disperse systems) in air. Coarser particles (coarse disperse system) settle in one-phase media and different types of flow can be evolved around a given particle during settling: therefore, different settling regimes must be distinguished. However, despite this fundamental phenomenon being widely examined in the literature, a generally accepted notation does not exist. Let's introduce a practical notation shown in Fig. 1.

The flow around the particle can be characterised based on the particle Reynolds number ($Re_x = V_o \cdot X \cdot \rho_l / \mu_l$). If the particle size is coarser than 10 μm, the dispersing medium typically can be treated as a continuum. In the 0.1–10 μm size range (V Cunningham – Millikan settling regime) both molecular collisions and mechanical forces (gravity, buoyancy, drag) affect the particle motion. The traditional calculation procedure in the III Transitional settling regime is iterative. In Fig. 1 the Kaskas equation (Tarján, 1997) is shown, but there are many equations to calculate the drag coefficient in this regime in the literature (Zhang et al., 2015; Wilson et al., 2006; Song et al., 2008). Wilson et al.

E-mail address: ejtfaitj@uni-miskolc.hu.

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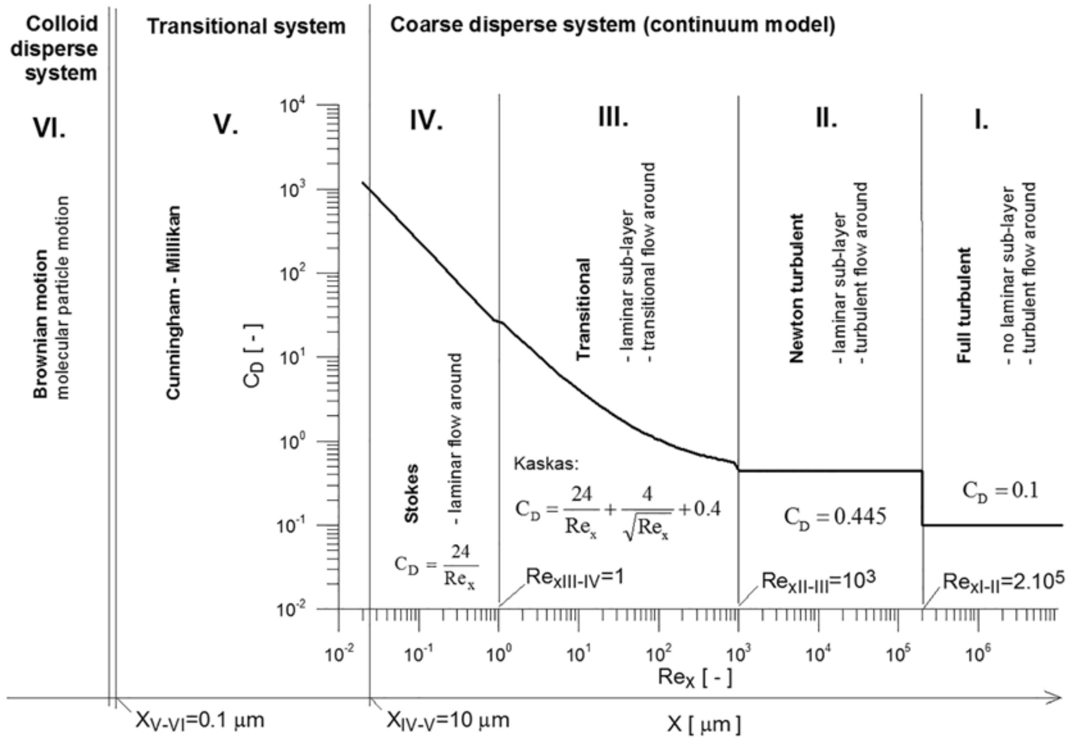

Fig. 1. The traditional calculation of the terminal settling velocity (spherical particle, incompressible Newtonian fluid, no-wall effect).

(2003) introduced a new diagram and calculation protocol for the continuum settling regimes (I–IV). The Wilson et al. calculation procedure is described in the Appendix A. Relationships between the traditional (Re_x–C_D) and the Wilson et al. (Re^* — V_o / V^*) parameters are described in the Appendix A as well. The recalculated and extended Wilson et al. terminal settling velocity diagram is shown in Fig. 2. The two calculation procedures result in almost identical terminal settling velocity values.

1.1.3. Single sphere settling in a non-Newtonian one phase medium

Enormous efforts have been made to describe the phenomenon of a single sphere settling in a non-Newtonian medium (Reynolds and Jones, 1989; Cho et al., 1992; D'Avino and Maffletone, 2015; Eltilib et al., 2011; Gheissary and van den Brule, 1996; Hariharaputhiran et al., 1998; Kelessidis, 2004; Omland, 2009). Generally speaking the calculation procedures are extremely complicated and they typically

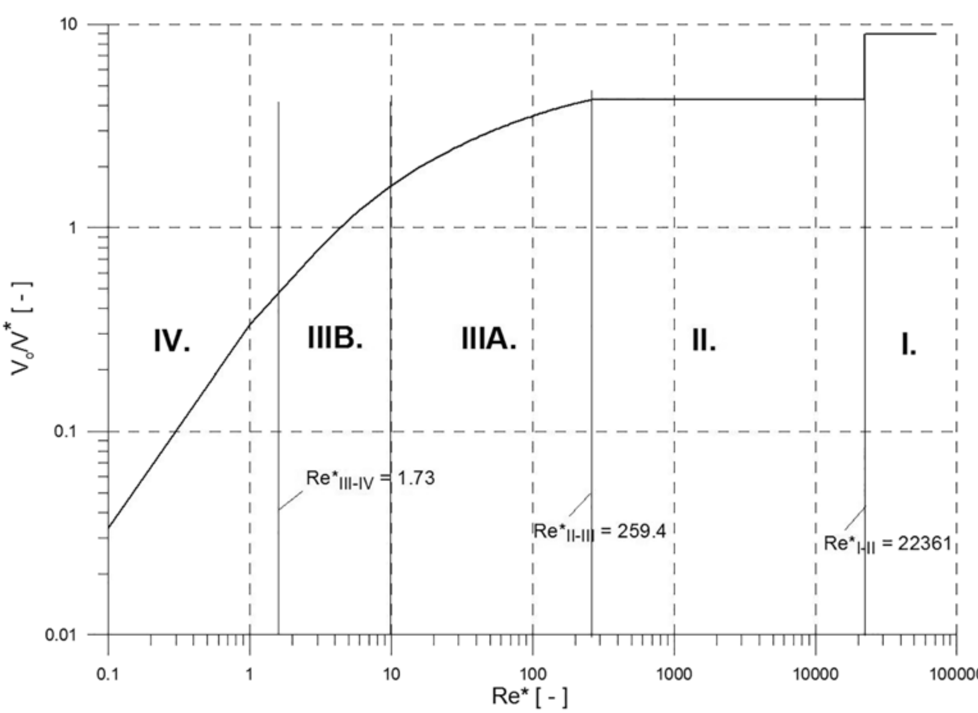

Fig. 2. The Wilson et al. (2003) calculation of the terminal settling velocity (spherical particle, incompressible Newtonian fluid, no-wall effect).