



Kernel functions to flotation bubble size distributions

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ABSTRACT

Flotation modelling has advanced from deterministic single particle-bubble models into using such models to solve flotation systems by using modern computational techniques. The step from a single particle- single bubble event to multiple events taking place in the large computational volume like a flotation cell poses the challenge of handling bubble and particle distributions in all computational cells.

The estimation of bubble size has either been omitted (constant size) or has been lately estimated by a population balance approach. The physical performance of flotation is excessively determined by the bubble size distribution (BSD). Therefore, the bubble size distribution estimate is crucial for modelling. Although the BSD can be measured, the underlying effects of different variables causing changes in break-up and coalescence rates producing changes in the measured BSD's are not well understood. This paper discusses the profound effects frothers have on both the coalescence and break-up of gas bubbles.

Depending on the bubble surface stiffness caused by frother adsorption, the drainage rate of fluid between two approaching bubbles is very different. Frothers like DF200 and Pentanol have a higher coalescence rate than frothers like DF250 and NF240.

Break-up is shown to be a function of the dynamic surface tension, not the static surface tension. Fast adsorbing frothers (DF200) have at very short time scales a higher rate of break-up.

The paper suggests a division of frothers into two distinct classes for modelling purposes. Those with fast adsorption and desorption, which leave the gas-air interface mobile and those frothers that by slower adsorption and desorption create stiff interfaces. The effects in real systems may be more varied. The modelling of subtler frother effects will not substantially improve modelling quality.

1. Introduction

Flotation modelling has the challenge to combine several physico-chemical phenomena into a concise model framework. The main body of modelling has been related to the well-known first order reaction model. During the years this approach with its additions and improvements has proven to be a good simple engineering model to be fitted with batch flotation data results *e.g.* a plug flow reactor.

$$\frac{dC}{dt} = -kC. \quad (1)$$

The rate constant k can be obtained experimentally for any given steady-state condition. However, to formulate the dependence of the rate constant from process variables has turned out to be difficult. There have been attempts to relate the rate constant to both local and global parameters (Jameson *et al.*, 1977) and to link the rate constant to the probabilistic bubble-particle encountering by the time averaged bubble

horizontal interface flux (termed bubble surface area flux S_b) (Gorain *et al.*, 1995a,b, 1996, 1999). To make the challenge more tractable, the total process has been generally divided into probabilistic sub-processes as outlined first by Gaudin (1932) and in more detail by Sutherland (1948). The “total probability of flotation” consists of the sub-process probabilities *e.g.* particle-bubble collision, attachment and stability (detachment)

$$k = P_j S_b. \quad (2)$$

The flotation rate k is then modelled as a product of the “total probability of flotation” and the frequency that bubbles and particles interact (come so close to each other for the above mentioned sub-processes to take place *e.g.* collide).

$$ZP_j = ZP_c P_a P_s = k. \quad (3)$$

There are several deterministic models for the particle-bubble collision (Gaudin, 1932; Sutherland, 1948; Yoon and Luttrell, 1989;

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Langmuir, 1948; Flint and Howarth, 1971; Dukhin, 1982; Dai et al., 2000). All these models, not repeated here, are considering a single bubble-particle pair. All of them show in general a relation between the collision frequency, particle (p) and bubble size (b) as

$$E_c = A \left(\frac{p}{b} \right)^n, \quad (4)$$

where parameter A varies from 3/2 in laminar to 3 in potential flows and n from 2 to 1, respectively.

A challenge has been how to expand the single bubble-particle pair models to handle large interacting bubble and particle populations.

Computational Fluid Dynamics (CFD) has become a versatile and indispensable tool to model any fluid flows containing devices and equipment, flotation devices among them. The challenges for meaningful flotation CFD are several, as the aim is to model two discrete poly-disperse phases interacting in a continuous fluid phase. Without discussing and detailing them, an important question to be asked is “What is the bubble size to be used in flotation modelling?”

A hypothesis to estimate bubble size has been to define a critical concentration from where added frother has no further effect on bubble coalescence (coalescence is prevented) and a minimum bubble size is reached (Cho and Laskowski, 2002). The minimum bubble size (b_0) is attributed to machine design and operation parameters but many authors highlighted that the minimum bubble size is also affected by frother type as it can influence the bubble break-up mechanism caused by turbulent eddies in high intensity zone (Chu et al., 2016; Jávora et al., 2013, 2016; Kracht and Finch, 2009). In industrial measurements of bubble size distributions Nasset et al. (2007) found cases with substantially differing distributions. Coalescence prevention as a determining factor of bubble size in flotation machines was questioned by Finch et al. (2008) using the data from the study of Nasset et al. (2007). However, the properties of the air/liquid interface are substantially determined by the rheological properties of the adsorbed layers (Fruhner et al., 1999) indicating possibilities of both bubble coalescence and break-up.

Population balance models (PBM) can be used in conjunction with CFD to estimate the local poly-disperse phase distributions, here the bubble number density functions (NDF) (Bhutani, 2016). The PBM conservation equation is

$$\frac{\partial n(b,x,t)}{\partial t} + \nabla \cdot ((ub)n) - \nabla \cdot (\bar{D}_x(b,x,t) \nabla n) = S_b(b,x,t), \quad (5)$$

where $n(b,x,t)$ is the number density function, b is the internal (bubble property, e.g. size) and x the external spatial coordinate respectively. The second and third terms on the left are the advective and diffusive parts of bubbles migrating (external coordinate space). The right-hand term is the source term describing all the processes taking place in the internal coordinate space

$$S_b = B_{br} - D_{br} + B_{co} - D_{co}, \quad (6)$$

where B_{br} and B_{co} are the birth functions due to bubble breakage and coalescence respectively. D_{br} and D_{co} are the respective death functions.

$$B_{br}(b) = \int_{\xi}^{\infty} m(b_1) a(b_1) c(b|b_1) n(b_1) db_1, \quad (7.1)$$

$$D_{br}(b) = a(b) n(b), \quad (7.2)$$

$$B_{co}(b) = \frac{1}{\delta} \int_0^{\infty} \left(\frac{b^2}{b'^2} \right) \beta(b', b_1) n(b') n(b) db_1, \quad (7.3)$$

$$D_{co}(b) = \int_0^{\infty} \beta(b, b_1) n(b) n(b_1) db_1. \quad (7.4)$$

Bubble breakage kernels $m(b_1)$, $a(b_1)$ and $c(b|b_1)$ define the number of bubbles produced in a break-up event, the frequency of break-up and the daughter distribution function respectively. Kernel $\beta(b|b_1)$ is the coalescence event frequency. This has also been divided into two parts, collision frequency and collision efficiency.

Population balance models can be solved either by the method of classes, where the NDF is discretized into a number of classes. Each class leads to an equation considering all the processes that will affect the units of the said class. The issue rising is mainly the bubble breakage and bubble coalescence. Each class would need its own kernel functions. The benefit is the natural reconstruct of the NDF. It is, however, computationally very expensive and time consuming to be used in flotation simulation. The other method to solve the bubble PBM is the method of quadrature of moments (QMOM) (McGraw, 1997; Marchisio and Fox, 2005; Bhutani, 2016). In the method the moments of the NDF are solved from the available transported moments. As is discussed by Bhutani (2016), for an approximate estimate of NDF four moments is often sufficient. For this approach a set of kernel functions would be sufficient. This, however, requires robust kernels.

Bubble break-up and coalescence kernels have been studied extensively for bubble columns (Prince and Blanch, 1990a; Luo and Svendsen, 1996; Martinez-Bazan et al., 1999, 2010; Lehr and Mewes, 2001; Wang et al., 2003; Zhao and Ge, 2007; Liao and Lucas, 2009, 2010; Solsvik and Jakobsen, 2015 among others). These studies have performed with electrolytes. They differ in several important aspects from flotation systems with varying chain length surface active reagents (frothers) and solids.

2. Bubble break-up kernels

The Martinez-Bazan et al. (2010) model assumes that a pair of bubbles can be formed, when the stresses caused by turbulence are larger than the stresses opposing deformation at the length scales corresponding to the mother bubble. There exists always a critical bubble size at each level of turbulent intensity that cannot be broken up. This critical size is expressed as

$$b_{crit} = \varepsilon^{-2/5} \left(\frac{12\lambda}{\beta\rho_s} \right)^{3/5}, \quad (8)$$

where ε is the turbulent energy dissipation, λ_s surface tension (without surfactant), ρ the density and β a constant. The probability of break-up is (Martinez-Bazan et al., 2010) for a bubble with a volume V is as follows

$$P_V(V^*) \propto \left(\frac{1}{2} \rho \beta (\varepsilon b_0)^{2/3} \right)^2 (V^{*2/9} - \Lambda^{5/3}) ((1-V^*)^{2/9} - \Lambda^{5/3}), \quad (9)$$

where b_0 is the mother bubble diameter, V its volume and Λ the ratio between the mother bubble size b_0 and the critical bubble size b_{crit} . The obtained bubble size distribution is (Martinez-Bazan et al., 2010):

$$f^*(b^*) = \frac{b^{*2} [b^{*2/3} - \Lambda^{5/3}] [(1-b^{*3})^{2/9} - \Lambda^{5/3}]}{\int_{b_{min}^*}^{b_{max}^*} b^{*2} [b^{*2/3} - \Lambda^{5/3}] [(1-b^{*3})^{2/9} - \Lambda^{5/3}] db^*}. \quad (10)$$

If surfactants are added to the solution in dilute concentrations their adsorption on the gas-water interphase will change the surface tension in a linear way

$$\lambda_s - \lambda = \Gamma^* R T, \quad (11)$$

where λ_s is the surface tension without the surfactant. Γ^* is the surface concentration of the surfactant (mass of surfactant per unit area of surface). Following the linear relationship, one can write for a small change in the surfactant to have an effect on the surface tension as follows (Stone and Leal, 1990)

$$\lambda^* - \lambda_s = (1 - \varphi). \quad (12)$$

This will change the critical bubble size of Eq. (8) to

$$b_{crit} = \varepsilon^{-2/5} \left(\frac{12\lambda_s(1-\varphi)}{\beta\rho} \right)^{3/5}. \quad (13)$$

As Martinez-Bazan et al. (2010) point out, the parameter Λ in Eq.

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