

# Effects of curvature and vorticity in rotating flows on hydrodynamic forces acting on a sphere



Toshiaki Fukada\*, Shintaro Takeuchi, Takeo Kajishima

Department of Mechanical Engineering, Osaka University, 2-1 Yamada-oka, Suita, Osaka 565-0871, Japan

## ARTICLE INFO

### Article history:

Received 2 August 2013

Received in revised form 5 October 2013

Accepted 11 October 2013

Available online 23 October 2013

### Keywords:

Particle rotation

Lift force

Free vortex

Forced vortex

Streamline curvature

Direct numerical simulation

## ABSTRACT

The angular velocity and the lift force on a spherical particle in rotating flows are studied by numerical simulation to investigate the effects of the curvature of the streamlines and the vorticity of the undisturbed background flow. The particle centre is fixed in space, and the rotating motion of the particle is studied in two types of rotating flows: free vortex (irrotational flow) and forced vortex (a rigidly-rotating flow). In both vortices, the angular velocity of the particle is found to exhibit self-similarity with respect to the curvature of the background flow in a range of particle Reynolds number between 5 and 100. Based on this finding, the angular velocity is represented, irrespective of the free and forced vortices, by a single correlation equation of the curvature, the vorticity and the particle Reynolds number. As for the lift force, the effect of the particle rotation induced by the background flow is non-negligible for both vortices. The lift force on a single freely-/non-rotatable particle in a free/forced vortex is found to be represented by linear combination of the following three effects; the streamline curvature and vorticity of the background flow, and the angular velocity of the particle rotation.

© 2013 Elsevier Ltd. All rights reserved.

## 1. Introduction

Solid-dispersed flows appear in many engineering applications as well as natural environment. In solid-dispersed turbulence containing particles of a density higher than the surrounding fluid, the turbulent flow causes non-uniform particle concentration known as the preferential concentration (Squires and Eaton, 1990; Eaton and Fessler, 1994) and the particle motion enhances the flow modulation. Understanding particle behaviour in vortical flows is important for modelling and controlling particle-laden flows.

To track a large number of particles in a practical simulation of particle-laden flow, point-particle models (e.g., Sommerfeld, 2003; Ren et al., 2011) have been often applied together with hydrodynamic force models acting on the particles. Some early drag force models for a sphere in a uniform flow were constructed based on experimental results (e.g. Schiller and Naumann, 1933; Morsi and Alexander, 1972). Also the lift force on a rotating sphere in a uniform flow (known as Magnus lift force) has been studied extensively. For very low particle Reynolds number (Re), Rubinow and Keller (1961) obtained the equation of Magnus lift force based on the Oseen approximation, and later, it was extended to the higher particle Reynolds numbers by experimental studies (Tanaka et al., 1990; Oesterlé and Dinh, 1998). As for the study on the lift force, Saffman (1965, 1968) derived the lift force on a stationary sphere

in a linear shear flow for  $Re \ll 1$ . Later, Mei (1992) developed the equations of Saffman lift force applicable to a wider range of the particle Reynolds number based on the results by Dandy and Dwyer (1990) and McLaughlin (1991). The lift force due to a linear shear flow decreases with the particle Reynolds number, and Kuriose and Komori (1999) showed that the shear-induced lift force changes the sign at around  $Re = 60$ . On the other hand, Sridhar and Katz (1995) showed in their experiment with microscopic bubbles that the lift force on a single bubble in a rotating flow is larger than those on a rigid particle in a linear shear flow cases. The result is consistent with the lift force in a rigidly rotating vortex (hereafter referred to as “forced vortex” as schematically shown in right hand side of Fig. 1) measured by van Nierop et al. (2007). The numerical result of Bluemink et al. (2008) indicates that the lift force on a sphere in a forced vortex increases with Re, which is the opposite trend from the cases of the linear shear flow. These previous works suggest that the lift force is sensitive to the difference of undisturbed background flows. Though the magnitude of the lift force is ordinarily smaller than that of the drag force on a sphere, the trajectories of the particles are strongly influenced by the lift force.

Particle rotation has a significant effect on the lift force, and the torque on the particle determines the particle rotation. The torque on a sphere and its angular velocity are often related as follows:

$$\mathbf{T} = \frac{\rho_f}{2} \left(\frac{D}{2}\right)^5 C_T \left| \frac{1}{2} \boldsymbol{\omega} - \boldsymbol{\Omega} \right| \left( \frac{1}{2} \boldsymbol{\omega} - \boldsymbol{\Omega} \right), \quad (1)$$

\* Corresponding author. Tel.: +81 668775111x3315.

E-mail address: [fukada@fluid.mech.eng.osaka-u.ac.jp](mailto:fukada@fluid.mech.eng.osaka-u.ac.jp) (T. Fukada).

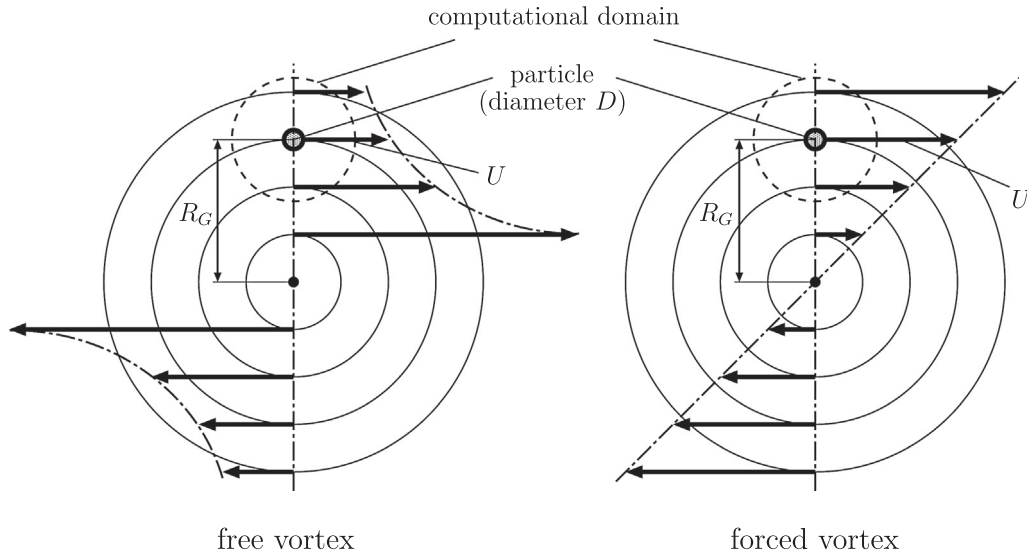


Fig. 1. Schematics of the free and forced vortices as the background flow.

where  $\rho_f$  is the density of the fluid,  $D$  the diameter of the particle,  $\omega$  the vorticity of the background flow,  $\Omega$  the particle angular velocity and  $C_T$  is the torque coefficient, which is often determined by the analytical result of a rotating sphere in a uniform flow for  $Re \ll 1$  (Rubinow and Keller, 1961) or the numerical result of a rotating sphere in a stationary fluid for a range of finite particle Reynolds numbers (Dennis et al., 1980). Eq. (1) assumes that the particle rotation is accelerated/decelerated until the particle angular velocity coincides with the half vorticity of the ambient flow. However, the freely-rotatable spherical particle adopts the angular velocity determined by the torque-free condition, and it is often different from  $\omega/2$ . Bagchi and Balachandar (2002a) suggested the following correlations for a sphere in a linear shear flow:

$$\Omega_{st} = \begin{cases} \frac{1}{2}(1 - 0.0364Re^{0.95})\omega & \text{for } 0.5 \leq Re \leq 5 \\ \frac{1}{2}(1 - 0.0755Re^{0.455})\omega & \text{for } 5 \leq Re \leq 200, \end{cases} \quad (2)$$

where  $\Omega_{st}$  is the steady angular velocity of the particle. Eq. (2) indicates  $|\Omega_{st}|/|\omega| < 0.5$  for the Reynolds numbers they studied, and satisfies the torque-free condition assumed in Eq. (1) only in the limit of creeping flow. On the other hand, Bluemink et al. (2008) simulated the particle behaviour in a forced vortex and they proposed the following correlation equation for the steady angular velocity of the particle:

$$\Omega_{st} = \frac{1}{2}(1 + 0.0045Re)\omega \quad \text{for } 5 \leq Re \leq 200. \quad (3)$$

Eq. (3) shows  $|\Omega_{st}|/|\omega| > 0.5$ , which is the opposite trend to Eq. (2). As Eqs. (2) and (3) show, the trend of  $\Omega_{st}$  depends on the background flow pattern.

Despite the fact that the spherical particle adopts different trends of  $\Omega_{st}$ - $\omega$  relation depending on the background flow, it is reported that lift forces on the sphere could be discussed with a linear combination of two independent contributions of shear and particle-rotation; Bagchi and Balachandar (2002a) found that, for a freely-rotating sphere in a linear shear flow, the shear-induced lift and Magnus-like lift (due to particle rotation in a uniform flow) could be decoupled in the Reynolds number range  $0.5 \leq Re \leq 200$ . Bluemink et al. (2010) further showed that, in a rigidly-rotating flow, the lift force induced by the flow and particle rotation can be superposed in the range  $5 \leq Re \leq 200$ . In addition, Bluemink et al. (2008) indicated that the cross-stream shear component of the background flow is a significant factor for the particle

rotation, and that a similar decoupling is applicable for  $\Omega_{st}$  in a range where the effects of shear components are relatively small. They also showed that the particle rotates in an irrotational flow such as a pure straining flow which is inconsistent with Eq. (1). Moreover, Bagchi and Balachandar (2002b) showed the asymmetry of the stress distribution on the surface of a stationary particle in a straining flow, which suggests a torque generation on the surface in an irrotational background flow.

The above correlations for the decoupled components are represented with the vorticity of the undisturbed background flow. However, the behaviour of the particle cannot be determined only by the effect of the vorticity. Considering that the vorticity vanishes in an irrotational flow, the effect of the (rotational/irrotational) rotating flow (on the angular velocity and lift coefficient of the particle) could be further separated into the contributions of the vorticity and the curved streamline. A free vortex is a typical example of a curved flow with no vorticity, and one of the simplest models of the turbulent vortex may be described as a rigidly-rotating core surrounded by a free vortex. The rotating flows of constant vorticities are characterised by the vorticity and streamline curvature, which is related to the cross-stream shear component of the rotating flows.

In this paper, we numerically study the hydrodynamic forces on a spherical particle in two different types of undisturbed background flow: a free vortex and a forced vortex. We propose an original convective boundary condition to deal with the background rotating flow within a finite domain size. The centre of the particle is fixed in space, and particles of freely-rotatable condition and non-rotatable condition are employed. Focusing on the rotating motion of the particle, the lift force and the torque acting on the particle are investigated, and the decoupling of the flow-induced lift and Magnus-like lift is discussed for both free and forced vortex cases by varying the geometric parameter of the vortices (i.e., curvature of the streamlines) and particle Reynolds number.

## 2. Governing equations and numerical method

### 2.1. Governing equations

The continuity and Navier–Stokes (N–S) equations are described on a non-inertial frame of reference fixed at the particle centre as follows:

$$\nabla \cdot \mathbf{u} = 0, \quad (4)$$

Download English Version:

<https://daneshyari.com/en/article/667254>

Download Persian Version:

<https://daneshyari.com/article/667254>

[Daneshyari.com](https://daneshyari.com)