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Modeling the macroscopic behavior of saturated deformable porous media using direct numerical simulations

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ABSTRACT

The behavior of saturated deformable porous media is investigated under compressive loading using direct numerical simulations. A parallel hybrid lattice Boltzmann and Finite Element method is employed to perform the numerical simulations. The efficiency and inherently parallel nature of LBM coupled with the robust FEM lends itself as a powerful tool to carry out the complex and computationally demanding direct numerical simulations of saturated deformable porous media.

Model porous media made up of spheres in a simple cubic arrangement are used in this work. The deformational response of such model geometries is compared to an existing analytical solution. For the comparison, the bulk properties of the macroscopic porous medium are obtained through the single phase properties of the constituents. The deformational behavior is seen to match the analytical solution closely. Thus it is found that the macroscopic behavior of a generic porous medium can be recovered through model porous medium constructed with simplified geometries. As a result, it is also seen that the LBM-FEM method is an accurate and suitable method for further investigations on porous media, particularly in nonhomogeneous and anisotropic deformable porous media.

Introduction

In the field of porous media flows, the application of direct numerical simulations (DNS) has mostly been limited to investigating the effect of geometric structure and flow parameters on hydraulic permeability, which is the measure of fluid conductance. Based on the structure of porous geometry, these investigations can be broadly classified as granular and fibrous media.

The research in granular media has been motivated by the need in soil mechanics and oil industry. Within granular geometries permeability studies have been performed on model geometry made up of spheres (Cancelliere et al., 1990; Bernsdorf et al., 2000), real porous geometries like soil samples and rock samples (Buckles et al., 1994; Soll et al., 1994; Manwart et al., 2002; Ferreol and Rothman, 1995) as well as geometries reconstructed stochastically based on porosity, specific surface area and two-point correlation (Biswal et al., 1999; Manwart and Hilfer, 1999). In fibrous porous media the permeability of cylinders in periodic arrangement (Happel, 1959; Sangani and Acrivos, 1981) as well as in random arrangement (Tahir and Tafreshi, 2009) has been extensively studied.

In the field of saturated deformable porous media the poroelasticity theory developed by Biot (1941, 1956) has been successfully used to predict the behavior of homogeneous porous media undergoing small deformation. A mathematically rigorous Theory of Porous Media (TPM) developed by Bowen (1980, 1982), de Boer (2005), Ehlers (1989), Ehlers and Eipper (1999), Ehlers and Markert (2000), Bluhm and de Boer (1997) has been applied to model both small and large deformations of porous media. In spite of the rigor and success, these methods make assumptions about the constitutive relationships, particularly those that govern the behavior of permeability. The assumption of a smeared medium adopted in these techniques requires that at any time particles from all the phases occupy a given position. Thus the governing equations contain material properties of the bulk medium as opposed to a single phase. Additional experimental work is needed in order to obtain such bulk media parameters.

Boutt et al. (2007) coupled the discrete element method with lattice Boltzmann method to model saturated porous media consisting of movable particles. However, the authors could not find any literature or data on utilizing DNS for understanding the behavior of saturated deformable porous media. Such investigations do not require the prior calculation of parameters like

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permeability or bulk modulus of the porous medium. With DNS there is no need to make any assumptions about the constitutive relationships (e.g. permeability and porosity) and thus it can capture the entire physics. DNS can be used as numerical experiments to validate and more importantly improve the existing mixture and poroelasticity model for saturated deformable porous media.

A fully coupled and parallelized numerical method using lattice Boltzmann method for fluid phase and finite element method for solid phase is employed for the analysis of saturated deformable porous media based on DNS. The lattice Boltzmann method with its simplicity and local nature of calculations is suitable to model complex flows (Aidun and Clausen, 2010) including porous media (Pan et al., 2006). The finite element method has been proven to be robust technique for modeling elastic deformations. The aim of the extensive code developed is to simulate the wet-pressing process occurring during paper making. Fig. 1 shows the schematic of the wet-pressing process where the wet paper and felt system passes through rollers which squeeze the water out of the system.

In this study, regular arrangements of spheres are used to develop a model porous medium. Section 'Method for modeling fluid–structure interactions in complex geometries' gives a brief description of the methodology applied and includes the validation of the implemented contact model with Hertz analytical solution for deformation of spherical surfaces during contact. Section 'de Boer, Ehlers and Liu Model' describes the analytical model given by de Boer, Ehlers and Liu for one-dimensional compression of a semi-infinite porous medium, based on the TPM. Section 'Behavior of simple cubic granular porous media' describes the model porous geometry setup using spheres. The section also describes the calculation of permeability and bulk elastic modulus followed by the discussion of results using geometries of granular arrangement. In Section 'Pore pressure: limitations of single-relaxation LBM', the predicted pore pressure inside the model geometry is compared to the analytical solutions and the limitations of the numerical method are discussed. The conclusions are provided in Section 'Conclusions'.

Method for modeling fluid–structure interactions in complex geometries

In this work a parallel hybrid lattice Boltzmann and finite element method is used to model the fluid–structure interaction in saturated porous media during deformation. Details of the method and its validation can be found in Khan and Aidun (2011). Here only a brief description of the method is provided.

The single relaxation D3Q19 implementation (Aidun et al., 1998; Aidun and Lu, 1995) of lattice Boltzmann method models the fluid phase. The accuracy of LBM has been compared to traditional numerical methods in fluid dynamics such as finite difference and finite volume methods (Bernsdorf et al., 1999; Sankaranarayanan et al., 2003). Moreover, the local nature of the calculations and the simple bounce-back scheme for no-slip condition make LBM a favorable tool for modeling single-phase flow through porous media (Noble et al., 1996), where the inherent parallel nature of LBM can be leveraged through distributed computing.

In order to model the solid deformations, a linear elastic finite element method is applied. The four-node tetrahedral element is used to discretize the geometry and solve the weak form of the Cauchy's equation. The Newmark's scheme forms the basis of marching forward in time.

The transfer of momentum between the fluid and solid phases is effected through the popular "link-bounce-back" scheme (Ladd, 1994; Aidun and Lu, 1995; Aidun et al., 1998). The boundary links as shown in Fig. 2 are identified using a ray tracing algorithm (MacMeccan et al., 2009). At each of these links a bounce-back boundary condition is applied to simulate a no-slip condition. The momentum (impulse) production term linking the discrete form of the fluid phase equations to the continuum based solid phase equations is given by

$$\delta \mathbf{P}_k = 2\mathbf{e}_k\left[f_k(\mathbf{x}, t_+) + \rho\frac{w_k}{3}\mathbf{v_b}\cdot\mathbf{e}_k\right], \tag{1}$$

where $f_k(\mathbf{x}, t_+)$ is the fluid distribution at $\mathbf{x}$ after the collision operation represented by t_+. $\mathbf{v_b}$ is the velocity of the boundary at the link, $\mathbf{e}_k$ is the direction along the boundary link, ρ is the density of the fluid and w_k are weights that depend on the direction k. For two-dimensional D2Q9 model $w_0 = 4/9$ for fluid particles at rest, $w_{1-4} = 1/9$ for fluid particles moving in non-diagonal directions, and $w_{5-8} = 1/36$ for diagonal directions. A simple linear interpolation scheme allows to transfer the momentum to the finite element nodes.

A near contact model developed by Ding and Aidun (2003) and later modified for elastic finite element particles by MacMeccan et al. (2009) is applied in this work. A link-wise repulsive force, based on the lubrication and contact forces, is applied to the interacting surfaces. The magnitude of the link-wise force is given as

$$\delta F_k = \begin{cases} 0 & \text{if } g > g_c, \\ \frac{3q}{2c_k^2\lambda}\nu^F\rho^F\left(\frac{1}{g^2} - \frac{1}{c_k^2}\right)\mathbf{U_a}\cdot\mathbf{e}_k & g_c < g < c_k, \\ \frac{3q}{2c_k^2\lambda}\nu^F\rho^F\left(\frac{1}{g^2} - \frac{1}{c_k^2}\right)\mathbf{U_a}\cdot\mathbf{e}_k + A_c\exp\left(\frac{-g+g_c}{\sigma_c}\right) & 0 < g < g_c, \end{cases} \tag{2}$$

where $\mathbf{U_a}$ is the approach velocity of the interacting surfaces, $\mathbf{e_k}$ is the direction vector of the lattice link and ν^F, ρ^F are the kinematic viscosity and density of the fluid. c_k is the length of the link k, g is the link-wise gap between surfaces, g_c is the contact cutoff distance, λ is the local surface curvature and q is chosen to be 0.6 (Ding and Aidun, 2003). The parameters g_c and σ_c are dependent on the surface roughness of the geometry and are determined *a priori*. A_c is chosen such that the repulsive contact force scales appropriately with the applied stresses (T_{zz}) i.e. $A_c \approx \frac{T_{zz}}{a_o}$ where a_o is the contact

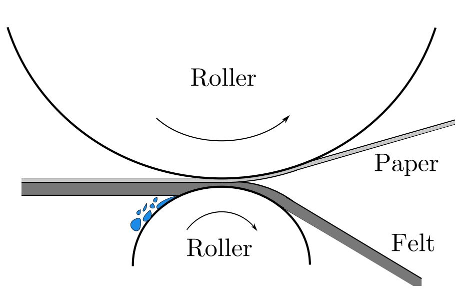

Fig. 1. Schematic of the wet-pressing process during paper making.

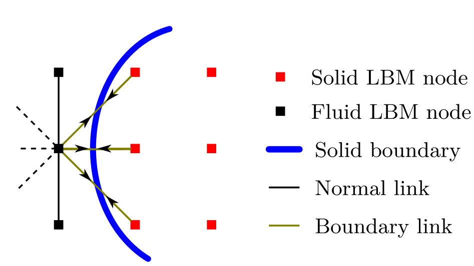

Fig. 2. "Link-bounce-back" scheme used for transfer of momentum between the fluid and solid phases.