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# Stochastic wind speed modelling for estimation of expected wind power output

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## HIGHLIGHTS

- Wind speed is modelled via an Ornstein–Uhlenbeck Geometric Brownian Motion model.
- Wind power statistics are calculated accurately with the Fokker–Planck equation.
- Expected capacity factors are computed and match those of empirical data.
- The proposed model shows a greater advantage over the Monte Carlo approach.

## ARTICLE INFO

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Stochastic differential equations
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Capacity factor
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## ABSTRACT

Increased wind energy penetration causes problems to the operation and system balancing of electric power systems. This in turn leads to the need for more detailed wind power modelling. The modelling and management of wind power involves two stages, neither of which is analytically tractable. In particular, the first stage involves stochastic variations in wind speed; wind speed typically presents noisy short-term variations, plus cyclicality over periods of 24 h and longer. The second stage refers to stochastic variations of the resulting wind power output, a non-linear function of wind speed. This paper proposes and tests an Ornstein-Uhlenbeck Geometric Brownian Motion model in continuous time to represent the wind speed, while including its longer-term daily cycle. It also illustrates a partial differential equation model of the wind speed and of the resulting wind power output, aiming at computing both their statistics. The proposed stochastic model has the potential to be used for various applications where wind speed or wind power are stochastic inputs, such as the optimal valuation of energy storage or system balancing. We verify by statistical tests that the results from the proposed model for the wind speed and the wind power match those from the empirical data of a wind farm located in Spain.

## 1. Introduction

In recent years wind energy has been increasingly integrated into electric power systems in order to meet renewable energy targets, aiming at reducing carbon emissions [1]. However, wind power is an intermittent, uncontrolled and inflexible type of generation [2]. Thus, although increased wind integration is essential to meet these targets, its volatile profile causes challenging system balancing problems [3]. This in turn leads to increased operating costs for system operators [4], associated with the ancillary service scheduling needed in order to back up wind power production [5]. Furthermore, wind operators may be exposed to the uncertainty of the imbalance markets, especially if any subsidies are removed [6]. Therefore, a more detailed and accurate modelling of wind speed and wind power is required, capturing not only the short-term variations but also any daily and seasonal cycles that wind may follow over different time periods and are important for effective operation [7].

Wind speed is stochastic in the short-term and its dynamics affect the dynamics of wind power, but on much longer time frames, such as days or months, wind displays daily (diurnal) and/or seasonal patterns essential for effective operation and planning of power systems [8–11]. Specifically, Ref. [8] states that daily cycles are site-specific with some sites and months exhibiting stronger daily cycles than others. Ref. [9] investigates how climate change conditions affect the diurnal wind power cycles of different sites and highlights that these changes must be evaluated in conjunction with the electrical load cycles to help utilities

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decide how they will handle excess wind penetration. Ref. [10] examines the impact of wind speed daily cycles on the location of wind parks and concludes that it is more important if these align with those of electricity demand consumption rather than selecting locations with the highest mean wind speed. Ref. [11] highlights that wind speed daily cycles affect the sizing of wind power plants but not significantly the size of individual wind turbines. This paper addresses wind speed and wind power fluctuations over intervals from minute to daily and seasonal time scales, which matter for both (optimal) operation and investment analysis [12].

The most common continuous Probability Density Function (PDF) for wind speed, used for fitting and modelling in renewable studies, is the Weibull distribution [13]. This has been utilised in Ref. [14] for wind energy resource estimation with the aim of guiding wind farm design decisions. In Ref. [15], it is used to generate scenarios used for wind power forecasting. It has also been used to obtain analytic expressions of capacity factors in Ref. [16] or of PDFs of wind power in Ref. [17]. However, wind speed can be represented effectively by the Weibull PDF only over extended time periods and not at daily, hourly or shorter time scales [18]. Specifically, because the Weibull PDF is not a time-dependent but a static distribution, it cannot represent the frequent short-term wind speed fluctuations that take place inside the hour and are important for trading and/or storing of wind energy [12]. In addition, it omits daily cyclicality in wind speed that varies the wind power output and affects the value of electricity produced [12].

Several stochastic models have been proposed for wind speed. One approach is to represent the stochastic process in discrete time through time series and auto-regressive moving average (ARMA) models, used mostly for modelling on hourly scales. For example, ARMA models are utilised in Ref. [7] for evaluating the impact of wind power in future power systems. They have also been used for wind speed modelling in reliability studies used in electrical power systems [19]. Ref. [20] examines four approaches based on ARMA models for forecasting of wind speed and of direction tuple. Ref. [21] gives an overview of the use of time-series models for wind speed or power forecasting, where it highlights that the prediction error increases with prediction time. In addition, we note here that second by second fluctuations in wind power output affect system balancing, and need continuous time models.

Hybrid models, i.e. artificial neural networks, including the back propagation, neural network, Elman neural network, and radial basis function neural network, combined with ARMA models, have been proposed for wind speed forecasting. In Ref. [22], a novel combined forecasting model for wind speed is developed on the basis of the variable weight combination theory. Ref. [23] proposes a multi-objective whole optimisation algorithm for wind speed forecasting that improves significantly the prediction accuracy compared to previous methods and also offers the possibility of multi-step wind speed predictions. A novel analysis-forecast system is proposed in Ref. [24] for short-term wind speed prediction that includes uncertainty analysis and mining of wind speed. Lastly, Ref. [25] develops a novel architecture of interval prediction of wind speed, which is also converted to wind power, and tests this new forecasting architecture over six multi-objective optimisation algorithms to prove significant improvements in performance.

For modelling short-term fluctuations of wind speed in continuous time, Stochastic Differential Equations (SDEs) have been used. The Langevin equation is used in Ref. [26] to generate wind speed fluctuations from three different PDFs over very short time scales (of less than 10 min). In Ref. [27], a novel approach is proposed to define the formulation of the drift and diffusion terms of an SDE used to model wind speed, based on a given stationary PDF and the autocorrelation of wind speed. In Ref. [28], fractional Brownian motion and multifractal random walk simulation are used to examine and quantify the statistical properties of short-term wind speed fluctuations. Ref. [29] proposes a methodology to produce statistically dependent wind speed scenarios that can be combined with stochastic programming decision models in wind power studies. SDEs have also been proposed recently for wind speed forecasting in Ref. [30] and wind power forecasting in Ref. [31]. Finally, Ref. [32] utilises SDEs for probabilistic wind power forecasting where it decomposes prediction errors into wind speed forecast errors and errors associated with the conversion from wind speed to wind power.

When modelling wind speed with a SDE, a partial differential equation (PDE), in particular the Fokker–Planck (FP) equation, is a candidate for generating the PDF at any given time [33]. The FP equation has been previously used in wind applications to derive stochastic dynamic models for representing the state variables of a wind turbine [34]. In Ref. [35], it is used to assess the uncertainty in the power output of variable-speed wind turbines. Finally, it has also been utilised to generate wind speed sequences and PDFs with the aim being to validate the results against empirical data [36].

Furthermore, the Ornstein–Uhlenbeck (O-U) stochastic process has frequently been utilised for wind modelling in power systems. It has been used to simulate real wind speed data [37], to examine the impact of different regulation strategies on the frequency of power systems [38] and to model wind power excess or deficit for wind storage applications in power systems [39]. Further, it is used in Ref. [18] to generate wind speed trajectories with statistical properties similar to those observed in an historical wind speed sample. In Ref. [40], it is used to develop probabilistic models for wind farm availability. Such a stochastic model has also been proposed in Ref. [41] to model wind power variations in short time frames (typically of hours) utilised for wind power representation. Similar model is used in Ref. [42] to represent the wind power output for real-time system balancing. Because of the short time scales examined, all the above models do not include seasonal or diurnal variations. Ref. [12] illustrates an energy storage model that can optimise smoothing and trading of wind energy, under the assumption that wind speed is modelled with an O-U model including its daily cycle. However, this is a ‘proof of concept’ model, and neither has been tested statistically, nor its parameters are based on empirical wind speed data. One of the aims of the present paper is to measure how well such a stochastic model fits real world wind data, over lead times varying from minutes to days. This constitutes an essential step towards testing the real-world usefulness of optimisation methods, such as those presented in Ref. [12].

Therefore, there exist in the literature many models for wind speed and power modelling but they lack the ability to capture in continuous time both the stochastic short-term variations and the daily cycle that wind speed follows along with the necessary statistical testing and empirical fitting of such stochastic processes based on real data.

In this paper, we propose an O-U Geometric Brownian Motion (GBM) model to represent wind speed variations. In the work presented herein, the log-wind speed is decomposed into a deterministic daily cycle and the O-U process; the latter is used to replicate short-term wind speed variations. The FP equation is used to compute the PDF of the wind speed and wind power is computed through a quadratic relationship with wind speed (power curve). Empirical fitting and validation of the stochastic model is done using real world time series of wind speed and wind power output. The four novel contributions of this paper compared to past work are summarised as follows:

1. The proposed stochastic modelling offers the advantage of a dynamic representation in continuous time for (random) wind speed variations. In addition, in contrast to previous literature [41,42], wind speed and power modelling are separated. The O-U GBM model is used to model wind speed directly, and wind power is obtained through a power curve mapping. This is very important and offers a realistic representation, since the PDF of the wind power and of the capacity factors (wind power output normalised to the maximum theoretical wind power output) is neither Gaussian nor log-normal because of the double-bounded nature of a power