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Natural element method for NON-GRAY radiation heat transfer problems

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A B S T R A C T

A radiation code based on the meshless Natural Element Method (NEM) for spatial discretization and on the Discrete Ordinates Method (DOM) for angular discretization of radiative transfer equation was developed for realistic calculation of the radiative heat transfer in two-dimensional complex geometries containing real gases with non-gray behavior (namely mixtures of H_2O, CO_2 and N_2). The spectral line-based weighted-sum-of-gray-gases (SLW) model was used to calculate the radiative properties of the medium. The boundaries are assumed to be opaque, diffuse and gray. The shape functions in the NEM are constructed by interpolations from the natural neighbors, and the essential boundary conditions can be imposed directly. The predictive accuracy of the proposed method is assessed by applying it to five test problems and comparing its predictions with other numerical solutions reported in the literature. The predictions are also compared with those of the gray gas model. A good agreement was observed between the results of the NEM/DOM/SLW method and other numerical methods, and it is shown that the natural element method has good accuracy, and can be used for the solution of radiative heat transfer problems in irregular enclosures containing non-gray media.

1. Introduction

Thermal radiation is an important heat transfer mode in many physical and engineering processes. Accurate modeling of high-temperature industrial systems, such as furnaces and combustors, in which thermal radiation is the predominant mode of heat transfer, necessitates reliable evaluation of the medium radiative properties and accurate solution of the radiative transfer equation (RTE).

The RTE is an integro-differential equation, and in the most general case involves seven independent variables, including three spatial coordinates, two angular directions, wavelength and time. Therefore, radiative heat transfer calculation is a challenging task, and exact analytical solutions of the RTE in general absorbing, emitting and scattering media are exceedingly difficult and explicit solutions are impossible for all but the simplest situations.

To tackle this problem, several numerical methods have been developed for radiative heat transfer calculations. These methods may be classified into integral or differential solution methods, although the Monte Carlo does not fit well into this classification. Differential solution methods need to account for the spatial, angular and spectral dependence of the radiation intensity. Several methods have been proposed for the spatial and angular discretization. As far as the angular discretization is concerned, the discrete ordinates method (DOM), finite volume method (FVM), and finite element method (FEM), among others, can be used. The DOM and FVM are probably the most widely used ones and have received significant attention and development due to their good accuracy, flexibility and moderate computational cost.

Several methods have been employed for the spatial discretization of the RTE, including the FVM, FEM, finite difference method and their variations. In the spatial discretization of a complex domain, severe mesh distortion may be difficult to avoid, and this may yield significant numerical errors when traditional methods, especially the FEM and FVM, are used. The meshless methods are alternative approaches to the mesh based numerical methods. In the meshless methods, the approximation of field variables is obtained using shape functions that are entirely

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constructed based on a group of discrete nodes, and no predefined nodal connectivity is required. The nodes are unstructured and distributed in the domain under consideration, and therefore can be freely moved, inserted and deleted. Many meshless methods have been developed. The common meshless methods mainly include the natural element method (NEM), the smooth particle hydrodynamics (SPH), the partition of unity finite element method (PUFE), the reproducing kernel particle method, the diffuse element method (DEM), the element-free Galerkin method (EFG), the meshless local Petrov-Galerkin method (MLPG) and their extensions.

Recently, meshless methods have been used to solve radiative heat transfer problems. The meshless local Petrov-Galerkin method based on discrete-ordinate equations was employed by Liu et al. [1,2] to solve radiative heat transfer in a one-dimensional slab [2] and the coupled radiative and conductive heat transfer problem in a slab with graded index media [1]. Liu et al. [3] applied this approach to two dimensional absorbing, emitting and scattering semitransparent graded index media. In these works, the moving least squares approximation was used to construct the shape functions. Tan and his coworkers [4–7] employed a least-squares collocation meshless method to solve radiative and combined radiative and conduction heat transfer in semitransparent media. Tan et al. [6] used a least-squares collocation meshless method based on the discrete ordinate equations to solve combined radiative and conductive heat transfer problems in 2D rectangular and cylindrical enclosures. A moving least-squares approximation was applied to construct the trial functions. Tan et al. [7] used the direct collocation meshless method to evaluate the accuracy and solution cost of the first/second-order radiative transfer equations. Luo et al. [8] employed the direct collocation meshless method with upwind scheme to improve accuracy and stability of the solution to the radiative transfer in strongly inhomogeneous media.

Sadat and his coworkers [9–13] used a moving least squares collocation meshless method (or a diffuse approximation) to solve the problems of radiation [9,11,12] and coupled radiation and conduction [10,13] heat transfer in regular and complex geometries. In order to avoid numerical oscillations, the even parity formulation of the discrete ordinates method was used. Wang et al. [14] used the meshless method presented in Ref. [9] to solve several purely radiative transfer problems in 2D and 3D geometries.

The natural element method is a meshless method based on the natural neighbor interpolants [15,16]. The NEM relies on the concepts of Voronoi diagrams and Delaunay triangulation to build Galerkin shape functions. The natural neighbor interpolants present some distinct and attractive features, including that: 1) imposing essential boundary conditions is easy, 2) shape functions are well-defined and robust approximation with no user-defined parameters on non-uniform grids, and 3) neighbor relationships are based on the local distribution and density of nodes at a given point. Zhang et al. [17–21] used the Galerkin natural element method, based on the discrete ordinates equation of radiative transfer, to solve several radiative and combined radiative-conductive heat transfer problems in irregular enclosures containing an absorbing, emitting and scattering medium. Zhang et al. [22] solved heat transfer problems in 2D semitransparent graded index media using the least-squares natural element method. The shape functions used in the NEM are constructed by the natural neighbor interpolations (Sibson interpolation and Laplace interpolation), which are strictly interpolants, and the essential boundary conditions can be imposed directly. It has been shown that the NEM is efficient, accurate and stable, and can be applied for solving radiative heat transfer problem in complex enclosures filled with semitransparent media.

Accurate modeling of radiative heat transfer also necessitates accurate estimation of radiative properties. This is a formidable task due to the extremely strong spectral dependence of radiative properties of participating gases present in the medium. The gray gas model is an approximation that assumes complete independence of radiative properties on wave number. Although this approach highly simplifies the RTE and its solution, the spectral radiative properties of a molecular gas vary so strongly and rapidly across the spectrum that the assumption of a gray gas is almost never a good one [23]. A wide variety of gas spectral radiative property models with different degrees of complexity and accuracy has been developed. These models can be grouped into line-by-line (LBL) model, narrow band (NB) models, wide band (WB) models and global models. Line-by-line calculations using spectral databases yield the highest accuracy possible, but are computationally very expensive. Hence, one needs to resort to more efficient and less computationally intensive methods, which – while less accurate than the line-by-line calculations – are considerably faster and yet sufficiently accurate for most engineering applications.

The weighted-sum-of-gray-gases (WSGG) model [24,25] is one of the global models and probably the most popular method to compute the gas radiative properties. However, in the case of significant temperature gradients, the WSGG model may yield important errors, higher than 30%, as discussed in Ref. [26]. The spectral line-based weighted-sum-of-gray-gases (SLW) model [27–31] is an improved version of the WSGG model, which has emerged as a promising alternative model able to provide high accuracy at moderate computational cost, and compatibility with any arbitrary solution method of the RTE. Denison and Webb [27,30,31] validated the model against LBL solutions on a wide variety of one and two-dimensional enclosures containing absorbing-emitting and scattering media. In the SLW model, the weights of the classical WSGG model are determined with the help of the absorption-line blackbody distribution function, which is calculated directly from the high resolution molecular spectrum of the gases. The integration of the RTE over wavenumber (wavelength) is then replaced by integration over absorption cross-section. The absorption-line blackbody distribution function permits efficient and accurate total radiative transfer calculations. Comparisons of the SLW model with various non-gray models are given in Refs. [32–34]. The comparison results revealed that the SLW model is the best choice with regard to computational time and accuracy.

Almost all practical situations involving radiative heat transfer occur in complex-shaped geometries, e.g., furnaces, boilers, combustion chambers, gas turbine combustors, spacecrafts and greenhouses, to name only a few. However, the vast majority of investigations in radiative heat transfer have concentrated on the study of radiative transfer in regular enclosures (most often rectangular enclosures). In addition, although most participating media display strong non-gray character, the vast majority of investigations to date have centered on the study of gray media. In addition, while radiative properties also generally depend strongly on temperature, concentration, etc., most calculations are limited to situations with constant properties, i.e., spectral radiative properties are assumed independent of spatial location and wavenumber. Several research works have been carried out to analyze non-gray radiation heat transfer, and in almost all of them the non-gray heat transfer problems have been solved within simple rectangular enclosures. Upon encouraging performance of