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Numerical assessment of the thermodynamic performance of thermoelectric cells via two-dimensional modelling ☆

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HIGHLIGHTS

- A 2-D model for thermoelectric cells is advanced based on the finite-volume method.
- The model accounts for the Fourier, the Thomson, and the Joule effects.
- The model predictions were validated against experimental data.
- The effects of the thermoelectric properties and the cell geometry were assessed.

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ABSTRACT

The present paper is aimed at putting forward a two-dimensional model for thermoelectric cells. The energy conservation equation was formulated in order to account for the Fourier (heat) conduction, the Thomson (thermoelectric) effect, and the Joule heating on the temperature distribution. The electric field was also modelled in order to come out with the current and voltage distributions. The governing equations were discretized by means of the finite-volume method, whereas the TDMA algorithm was adopted for solving the sets of linear equations. An explicit solution scheme was employed to address the temperature influence on the thermoelectric effect. The model results have been compared with experimental data, when a satisfactory agreement was achieved for both cooling capacity and COP, with errors within a 10% band. In addition, the model was employed to assess the effects of the thermophysical properties and the couple geometry on the thermodynamic performance of the thermoelectric cell.

1. Introduction

In the past decades, solid-state cooling technologies have come onto some particular market niches, especially the applications related to portable cooling [1]. The most significant advances have been achieved in the realm of thermoelectric cooling [2], in which an electric current produces a temperature difference in a couple of dissimilar semiconductor materials. A typical thermoelectric module is manufactured with two thin ceramic wafers and an array of p- and n-type blocks of doped semiconductor material sandwiched between them. A pair of p- and n-type blocks connected electrically in series and thermally in parallel make up a thermoelectric couple [3].

Several studies have been conducted both theoretically and numerically to assess the thermodynamic performance of thermoelectric cells. Samples of the most influencing works are summarised in Table 1. The literature review points out that most models are one-dimensional, being not able to evaluate the influence of the cell geometry on its performance. In addition, the literature analysis also reveals that the few available multidimensional approaches are often developed aided by commercial packages, which provide restricted access to the mathematical formulation and the numeric algorithm. At last, most models do not account for the heat transfer in the air cavity, which may also affect the system performance. The present paper is therefore aimed at advancing a tailor-made two-dimensional model, in the realm of the irreversible thermodynamics, which is suitable to evaluate the sensitivity of the thermophysical properties and the cell geometry on its thermodynamic performance.

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Nomenclature

Roman

COP	coefficient of performance ($W\,W^{-1}$)
j	electric current density ($A\,m^{-2}$)
k	thermal conductivity ($W\,m^{-1}\,K^{-1}$)
L_x	width (m)
L_y	height (m)
L_z	length (m)
m	number of control volumes (y-direction)
n	number of control volumes (x-direction)
N	number of thermoelectric couples in the cell
Nu	Nusselt number (dimensionless)
$\dot{q}$	volumetric heat generation rate ($W\,m^{-3}$)
q	heat flux ($W\,m^{-2}$)
$\dot{Q}$	heat transfer rate (W)
T	temperature (K)
V	voltage (V)
Z	figure-of-merit (K^{-1})

Greek

α	Seebeck coefficient ($V\,K^{-1}$)
γ	electrical conductivity ($A\,V^{-1}\,m^{-1}$)
φ	generic variable
ρ	electrical resistivity ($Vm\,A^{-1}$)
τ	Thomson coefficient ($V\,K^{-1}$)
$\hat{\Psi}$	dimensionless response variable
λ	coefficients of Eq. (27)
$\hat{\varphi}$	dimensionless values of φ

Subscripts

c	cold end
e, w, n, s	control surfaces
P, E, W, N, S	control volumes
h	hot end

2. Mathematical model

A thermoelectric cell is comprised of several couples of p and n semiconductors connected electrically in series and thermally in parallel, and separated from each other by a cavity filled with air. In the present work, the physical model is restricted to a thermoelectric couple, as illustrated in Fig. 1, which in turn is subdivided into ten domains, as summarised in Table 2. The dimensions in Table 2 refer to the thermoelectric device under analysis [13], which has been taken as reference for the present study given its application to portable coolers [14].

The mathematical model is based on the following key assumptions: (i) steady-state two-dimensional approach, (ii) thermoelectric properties regarded as functions of the temperature only, (iii) negligible internal contact resistances (both thermal and electric), (iv) both n and p elements have the same Seebeck coefficient, but with different signs, and (v) heat transfer by both advection and radiation are disregarded, so that $Nu = 1$ in the cavity. Thus, a local energy balance yields,

$$\vec{\nabla}\cdot\vec{q} = \dot{q} \tag{1}$$

where $\dot{q}$ is the rate of heat generation, and the heat flux, $\vec{q}$, is calculated from the following fundamental relation obtained from the irreversible thermodynamics [15]:

$$\vec{q} = -k\vec{\nabla}T + \alpha T\vec{j} \tag{2}$$

where first term on the right-hand side stands for the heat conduction (referred hereafter as Fourier effect), where k is the thermal conductivity, and the second term is associated with the Seebeck effect, where α is the Seebeck coefficient. The divergent of Eq. (2) yields,

$$\vec{\nabla}\cdot\vec{q} = -\vec{\nabla}\cdot\left(k\vec{\nabla}T\right) + T\vec{j}\cdot\vec{\nabla}\alpha + \alpha\vec{j}\cdot\vec{\nabla}T + \alpha T\left(\vec{\nabla}\cdot\vec{j}\right) \tag{3}$$

where $\vec{\nabla}\cdot\vec{j} = 0$ at steady-state conditions to ensure the continuity of the current density. In addition, the definition of electric field yields,

$$-\vec{\nabla}V = \rho\vec{j} + \alpha\vec{\nabla}T \tag{4}$$

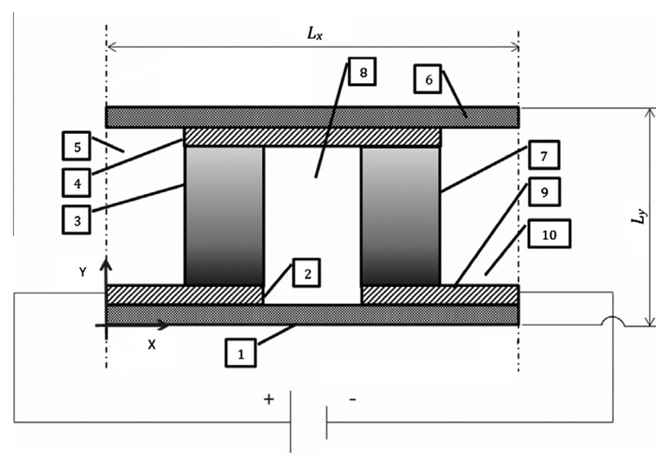

Fig. 1. Schematic representation of the physical model.

Table 1
Recent literature on performance assessment of thermoelectric cells.

Author	Year	Approach	Thomson effect	Cavity convection	Cavity radiation	Physical domain	Properties as $f = f(T)$
Huang et al. [4]	2005	Analytical	Yes	Yes	Yes	1D	No
Pramanick and Das [5]	2006	Analytical	Yes	No	No	1D	No
Lee and Kim [6]	2006	Numerical	No	No	No	1D	No
Yamashita [7]	2009	Analytical–experimental	Yes	No	No	1D	Yes
Chen et al. [8]	2011	Numerical	Yes	Yes	Yes	3D	No
Meng et al. [9]	2011	Numerical	Yes	Yes	Yes	1D	Yes
Du and Wen [10]	2011	Numerical–experimental	Yes	No	No	1D	Seebeck only
Chen et al. [11]	2012	Numerical	Yes	No	No	3D	Seebeck only
Pérez-Aparicio et al. [12]	2012	Numerical	Yes	Yes	Yes	3D	Yes