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Optimal charging of an electric vehicle using a Markov decision process

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HIGHLIGHTS

- This paper proposes an algorithm to optimally charge an electric vehicle considering the usage of the vehicle.
- The charging policy depends on the use of the vehicle, the risk aversion of the end-user, and the electricity price.
- The model is versatile and can easily be adapted to any specific vehicle, thus providing a customized charging policy.

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ABSTRACT

The combination of electric vehicles and renewable energy is taking shape as a potential driver for a future free of fossil fuels. However, the efficient management of the electric vehicle fleet is not exempt from challenges. It calls for the involvement of all actors directly or indirectly related to the energy and transportation sectors, ranging from governments, automakers and transmission system operators, to the ultimate beneficiary of the change: the end-user. An electric vehicle is primarily to be used to satisfy driving needs, and accordingly charging policies must be designed primarily for this purpose. The charging models presented in the technical literature, however, overlook the stochastic nature of driving patterns. Here we introduce an efficient stochastic dynamic programming model to optimally charge an electric vehicle while accounting for the uncertainty inherent to its use. With this aim in mind, driving patterns are described by an inhomogeneous Markov model that is fitted using data collected from the utilization of an electric vehicle. We show that the randomness intrinsic to driving needs has a substantial impact on the charging strategy to be implemented.

1. Introduction

Electric vehicles (EVs) are emerging as a sustainable and environmentally friendly alternative to conventional vehicles, provided that the energy used for their charging is obtained from renewable energy sources. The energy generated from renewable sources such as sunlight, wind and waves is, however, dependent on weather conditions. As a consequence, the electricity production from these sources is inherently uncertain in time and quantity. Furthermore, electricity has to be produced and consumed at the same time, as the large-scale storage of the energy generated is, still today, very limited. As a result, the energy obtained from renewables may be wasted in times when the demand for electricity is not high enough to absorb it, with a consequent detrimental effect on the profitability of renewables. Since the battery in an EV is basically a storage device for energy, the large-scale integration of EVs in the transportation sector may contribute to substantially increasing the socioeconomic value of an energy system with a large renewable component, while reducing the dependence of the transportation sector on liquid fossil fuel.

For this reason, EVs have received increased interest from the scientific community in recent years (detailed literature reviews of the state of the art can be found in [1,2]). Special attention has been given to the analysis of the effect of EVs integration on the electricity demand profile [3,4], emissions [5] and social welfare [6–8], and to the design of charging schemes that avoid increasing the peak consumption [9,10], help mitigate voltage fluctuations and overload of network components in distribution grids [11], and/or get the maximum economic benefit from the storage capability of EVs within a market environment, either from the perspective of a single vehicle [12,13] or the viewpoint of an aggregator of EVs [14,15]. In all these publications, though, and more generally in the technical literature on the topic, the charging problem of an EV is addressed either by considering deterministic driving patterns, when the focus is placed on the management of a single vehicle, or by aggregating the driving needs of different EV

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users, when the emphasis is on modeling a whole fleet of EVs. This aggregation, however, obscures the dynamics of each specific vehicle. Likewise, the deterministic driving patterns of a single EV are often based on expected values or stylized behaviors, which fail to capture important features of the charging problem such as the daily variation in the use of the vehicle or potential user conflicts in terms of not having the vehicle charged and ready for use. A stochastic model for driving patterns provides more insight into these aspects and becomes fundamental for applying a charging scheme in the real world. Despite this, the stochastic modeling of driving patterns has received little attention from the scientific community, as pointed out in [1]. We mention here the research work by [16], in which they aim to capture the uncertainty intrinsic to the vehicle use by means of a Monte Carlo simulation approach. They assume, however, an uncontrolled charging scheme.

The work developed in this paper departs from the following two premises:

1. The primary purpose of the battery of an EV is to provide power to drive the vehicle and not to store energy from the electricity grid. Consequently, it is essential that enough energy is kept in the battery to cover any desired trip. This calls for a decision tool that takes into account the driving needs of the EV user to determine when charging can be postponed and when the battery should be charged right away.
2. The complexity of human behavior points to a stochastic model for describing the use of the vehicle. In turn, this stochastic model should be integrated into the aforementioned decision tool and exploited by it.

That being so, this paper introduces an algorithm to optimally decide when to charge an EV that exhibits a stochastic driving pattern. The algorithm builds on the inhomogeneous Markov model proposed in [17] for describing the stochastic use of a single vehicle. The model parameters are then estimated on the basis of data from the use of the specific vehicle. The approach captures the diurnal variation of the driving pattern and relies only on the assumption that the EV-user's driving habits can be explained and modeled as a stochastic process, more particularly, as an inhomogeneous Markov chain. This makes our modeling approach noticeably general and versatile. Our algorithm thus embodies a *Markov decision process* which is solved recursively using a stochastic dynamic programming approach. The resulting decision-support tool allows for addressing issues related to charging, vehicle-to-grid (V2G) schemes [12,18], availability and costs of using the vehicle. The algorithm runs swiftly on a personal computer, which makes it feasible to implement on an actual EV.

The remainder of this paper is organized as follows: In Section 2 the stochastic model for driving patterns developed in [17] is briefly described, tailored to be used in the present work, and extended to address the problem of driving data limitations through hidden Markov models. Section 3 introduces the algorithm for the optimal charging of an EV as a Markov decision process that is solved using stochastic dynamic programming. Section 4 provides results from a realistic case study and explores the potential benefit of implementing V2G schemes. Section 5 concludes and provides directions for future research within this topic.

2. A stochastic model for driving patterns

In this section we summarize and extend the stochastic model for driving patterns developed in [17]. We refer the interested reader to this work for a detailed description of the modeling approach.

2.1. Standard Markov model

A state-space model is considered to describe the use of the EV. In its simplest form, it contains two states, according to which the vehicle is either *driving* or *not driving*. A more extensive version of the model would include a larger number of states which could capture information about where the vehicle is parked, how fast it is driving or what type of trip it is on. The basics of the general multi-state stochastic model are described in this section, including how to fit a specific model on an observed data set.

Let X_t, where $t \in \{0, 1, 2, \dots\}$, be a sequence of random variables that takes on values in the countable set S, called the state space. Denote this sequence as X. We assume a finite number, N, of states in the state space. A Markov chain is a random process where future states, conditioned on the present state, do not depend on the past states [19]. In discrete time X is a Markov chain if

$$\mathbb{P}(X_{t+1} = k | X_0 = x_0, \dots, X_t = x_t) = \mathbb{P}(X_{t+1} = k | X_t = x_t) \quad (1)$$

for all $t \geqslant 0$ and all $\{k, x_0, \dots, x_t\} \in S$.

A Markov chain is uniquely characterized by the transition probabilities, $p_{jk}(t)$, i.e.

$$p_{jk}(t) = \mathbb{P}(X_{t+1} = k | X_t = j). \quad (2)$$

If the transition probabilities do not depend on t, the process is called a homogeneous Markov chain. If the transition probabilities depend on t, the process is known as an inhomogeneous Markov chain.

When it comes to the use of a vehicle, it is appropriate to assume that the probability of a transition from state j to state k is similar on specific days of the week. Thus, for instance, Thursdays in different weeks will have the same transition probabilities. For convenience we further assume that all weekdays (Monday through Friday) have the same transition probabilities. In other words, we consider that the transition probabilities of the inhomogeneous Markov chain vary within the day, but not from day to day. These assumptions can be easily relaxed or interchanged with other assumptions and as such, are not essential to the model. With a sampling time in minutes, and taking into account that there are 1440 min in a day, this leads to the assumption:

$$p_{jk}(t) = p_{jk}(t + 1440). \quad (3)$$

This assumption implies that the transition probabilities, defined by (2), are constrained to be a function of the time, s, in the diurnal cycle. Let the matrix containing the transition probabilities be denoted by $\mathbf{P}(s)$. This matrix characterizes the driving pattern of the specific vehicle under consideration using N states. It has the form:

$$\mathbf{P}(s) = \begin{pmatrix} p_{11}(s) & p_{12}(s) & \dots & p_{1N}(s) \\ p_{21}(s) & p_{22}(s) & \dots & p_{2N}(s) \\ \vdots & \vdots & \ddots & \vdots \\ p_{N1}(s) & p_{N2}(s) & \dots & p_{NN}(s) \end{pmatrix}, \quad (4)$$

where $p_{jj}(s) = 1 - \sum_{i=1, i \neq j}^{N} p_{ji}$.

Now let $n_{jk}(s)$ define the number of observed transitions from state j to state k at time s. From the conditional likelihood function, the maximum likelihood estimate of $p_{jk}(s)$ for the inhomogeneous Markov chain can be found as:

$$\hat{p}_{jk}(s) = \frac{n_{jk}(s)}{\sum_{k=1}^{N} n_{jk}(s)}. \quad (5)$$

A discrete time Markov model can be formulated based on the estimates of $\mathbf{P}(1), \mathbf{P}(2), \dots, \mathbf{P}(1440)$. One apparent disadvantage of such a discrete time model is its huge number of parameters, namely $N \times (N - 1) \times 1440$, where $N \times (N - 1)$ parameters have