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Dynamic instability of rotating doubly-tapered laminated composite beams under periodic rotational speeds

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ABSTRACT

Dynamic instability analysis of doubly-tapered cantilever composite beams rotating with periodic rotational velocity is conducted in the present work for out-of-plane bending (flap), in-plane bending (lag) and axial vibrations. Rayleigh-Ritz method and classical lamination theory are used along with an energy formulation. Bolotin's method is applied to obtain the instability regions. To verify the dynamic instability analysis results, time responses are investigated at different locations of the instability region by using the Runge-Kutta method. A comprehensive parametric study is conducted in order to understand the effects of taper configurations and various system parameters including mean rotational velocity, hub radius, double-tapering angles and stacking sequences, on the dynamic instability characteristics of the composite beam. The composite material considered in the present work in numerical results is NCT-301 graphite-epoxy prepreg.

1. Introduction

Composite material has outstanding engineering properties, such as high strength/stiffness to weight ratios and favorable fatigue characteristics and due to this reason composite material is used in the design of rotating structure such as aircraft turbo fans, helicopter rotor blades and wind turbine blades. In some specific applications such as helicopter blades, robot arms, turbine blades and satellite antenna components need to be stiff at one location and flexible at another location. A typical example is a helicopter rotor blade, where a progressive variation in the thickness of the blade is required to provide high stiffness at the hub and flexibility in the middle of blade length, to accommodate for flapping. This type of structure is formed by terminating or dropping off plies at the pre-determined location to reduce the stiffness of the structure which is called tapered composite structure [1]. These elastic tailoring properties and more significant weight saving than commonly used laminated components allow an increasing use of tapered composite structure in commercial and military aerospace and power generation engineering applications.

In a rotating composite beam, dynamic instability can be caused by in-plane periodic load or by periodic rotational velocity. When the frequency of dynamic periodic load and the frequency of free vibration of the component coincide, parametric resonance will occur in the structure, which results dynamic instability of the structure. Mechanical structures that operate within the instability region will experience parametric resonance. This incident reduces the durability of structure and leads to unpredictable and catastrophic failure. Especially in an aircraft engine or in wind turbine, rotating blade experiences periodic aerodynamic loads which change the constant angular velocity to pulsating angular velocity. The excitation frequency of the pulsating load may coincide with the natural frequency of free vibration of the blade and the blade becomes dynamically unstable from nominal position. Even when the parametric vibration might not have an immediate effect, it is a future threat for fatigue failure, if they continue to act. Dynamic instability analysis introduces a method to predict and prevent the parametric vibration which is necessary to design a structure for safety and reliability especially when it is out of immediate maintenance.

Dynamic instability analysis of a beam subjected to periodic loads is an important and advanced research topic. A number of research works can be traced to parametric resonance or dynamic instability of isotropic non-rotating beam. Bolotin [2] first comprehensively reviewed the research works on dynamic instability problems of bars, plates and shells. Hyun and Yoo [3] studied the dynamic stability of an axially oscillating cantilever beam considering the stiffness variation. The dynamic stability of a radially rotating beam subjected to base excitation was investigated by Tan et al. [4].

With a few exceptions, most of these studies have addressed the axially oscillating problem. On the other hand, Yoo et al. [5] analyzed the dynamics of a rotating cantilever beam. They presented a linear modeling method for the dynamic analysis of a flexible beam undergoing overall motion. Based on this modeling method, Chung and Yoo

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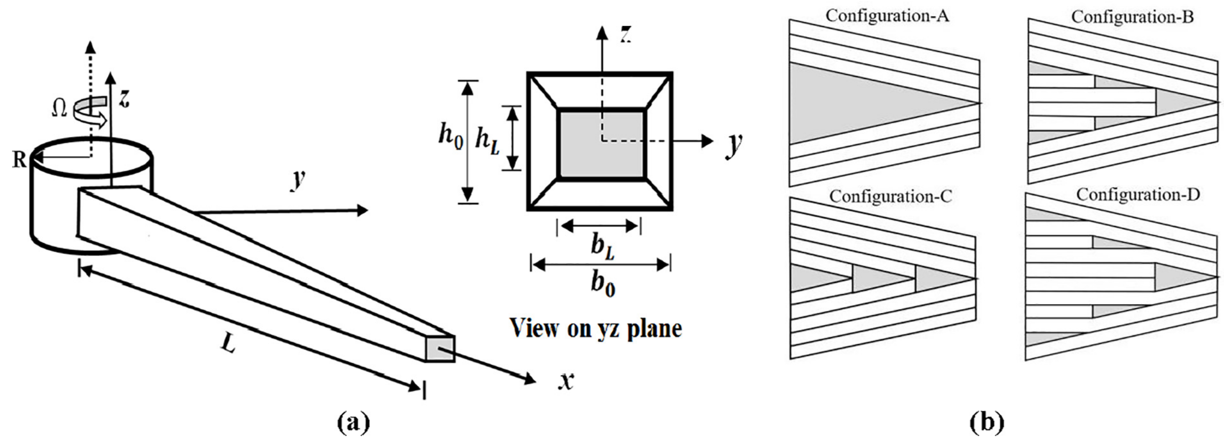

Fig. 1. (a) Doubly-tapered rotating composite beam (b) Different taper configurations.

[6] derived the partial differential equations of motion for a rotating cantilever beam and discretized using the Galerkin method to investigate the natural frequencies and time response. This study investigates the dynamic stability of the flap wise motion of a cantilever beam by using the method of multiple scales, when the beam oscillates in the rotational direction.

In relation to composite materials, Saravia et al. [7] first investigated the dynamic stability behavior of thin-walled rotating composite beams using finite element method. Lin and Chen [8] studied the dynamic stability of a rotating composite beam with a constrained damping layer subjected to axial periodic loads. Chen et al. [9] investigated the dynamic stability of rotating composite shafts under axial periodic loads. Chattopadhyay and Radu [10] studied the dynamic instability of composite laminates using a higher order theory.

In addition to these works, many researchers considered the dynamic instability of beams that are subjected to follower forces. Beck [11] examined the dynamic instability of a cantilever beam subjected to an axial follower force that was applied at the free end. Instability of a rotating cantilever beam subjected to dissipative, aerodynamic, and transverse follower forces has been investigated by Anderson [12]. Most recently, Torki et al. [13] evaluated the stability characteristics of cantilevered FGM cylindrical shell under axial follower forces. They have used Love's hypothesis to derive the differential equations of motion, and used an extended Galerkin's method to solve the equations of motion. Goyal et al. [14] and Kim et al. [15] studied the dynamic stability of laminated composite beams subjected to non-conservative tangential follower loads.

To the present authors' knowledge, a comprehensive study on the dynamic instability of doubly-tapered (thickness-and width-tapered) rotating composite beam has not so far been carried out. In the present paper, the dynamic instability of doubly-tapered composite beam rotating with periodic rotational velocity is investigated considering out-of-plane bending, in-plane bending and axial undamped vibrations. Rayleigh-Ritz approximate solution method based on classical lamination theory has been employed for energy formulations. Bolotin's method is applied to obtain the instability regions. A comprehensive parametric study is conducted in order to understand the effects of various parameters including mean rotational velocity, hub radius, double-tapering and different stacking sequences. In addition, to verify the instability analysis results, time responses are investigated at different locations of the instability region by using the Runge-Kutta method.

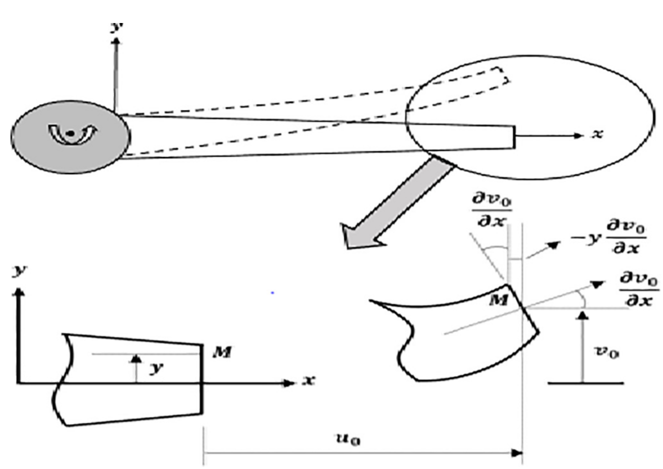

Fig. 2. Deformation of the beam in the lamination plane (x-y plane).

2. Energy formulation

Consider a laminated composite beam of length L, which is attached to a hub of radius R, as shown in the Fig. 1 in Cartesian coordinates. The hub rotates about its axis at a constant angular speed Ω rad/s. The origin for the coordinates is taken at the edge of the hub. The x-axis coincides with the neutral axis of the beam, the z-axis is parallel to the axis of rotation and the y-axis lies in the plane of rotation.

View on y-z plane illustrates beam changing the thickness from h_0 to $h_L$and changing its width from b_0 to b_L over the length L. The laminated composite beam consists of N layers, numbered from the lower to the upper face. To study the out-of-plane bending vibration, x-y plane is chosen as the mid-surface and reference plane. Dynamic instability analysis of the above composite beam requires associated equation of motion. The Lagrange's equation can be used to obtain the equation of motion of this physical system. To use Lagrange's equation, total strain energy, including work done by the centrifugal force and kinetic energy of the system, needs to be determined. Considering that the beam's length to thickness ratio is high, Classical Laminate Theory (CLT) can be used to determine the strain energy which assumes that transverse shear strains are zero and neglects z-direction stress, that is $\sigma_{zz}^k = 0$, and$\gamma_{xz}^k = \gamma_{yz}^k = 0$ for the k-th ply. Therefore, strain energy for a laminate with N plies can be written as:

$$U = \sum_{k=1}^{N} \frac{1}{2} \int\int\int (\sigma_{xx}^k \varepsilon_{xx}^k + \sigma_{yy}^k \varepsilon_{yy}^k + \tau_{xy}^k \gamma_{xy}^k) dx\, dy\, dz \tag{1}$$

where σ_{xx}^k and σ_{yy}^k denote the stresses in corresponding ply along the x and y directions, respectively, ε_{xx}^k and ε_{yy}^k denote the strains in corresponding ply along x and y directions, respectively. τ_{xy}^k is shear stress and γ_{xy}^k is shear strain in the corresponding ply acting on the x-y plane. For the doubly-tapered laminated composite beam shown in the Fig. 1, strain energy equation can be written as: