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Geometrically nonlinear bending analysis of functionally graded beam with variable thickness by a meshless method

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ABSTRACT

Geometrically nonlinear bending deformation of Functionally Graded Beams (FGBs) with variable thickness is simulated by a meshless Smoothed Hydrodynamic Particle (SPH) method. The material properties of FGB is assumed to be varied smoothly in the thickness according to exponent-law distribution. To prevent the mesh-distortion in element-based numerical method, meshless SPH method is adopted, where corrective smoothed particle method and total-Lagrangian formulation are employed to improve its precision and stability. To validate the present SPH method, several numerical examples are performed and compared to analytical and finite element solutions.

1. Introduction

Since introduced in thermal barrier on the space shuttle by Japanese scientists in 1984, Functionally Graded Materials (FGMs) have drawn a great attention of scientists and engineers with wide application in aerospace, automotive, civil, electronics, military, etc. FGMs possesses noticeable advantage in the smooth gradation of material properties which can reduce the residual stresses and stress concentrations. This makes FGMs different from traditional laminated composites in which the material properties is abruptly changed across the physical layer interface leading to large interlaminar stresses and delamination defect. FGMs can be fabricated by desired constituents with designed volume fractions to precisely control its' properties for engineering applications.

With the increasing structural application of FGMs, numerous studies to predict the mechanical response have been performed through analytical and numerical methods. The thermo-mechanical behavior of a functionally graded extensible Timoshenko beam is studied by employing the simplified and original boundary conditions and solving the nonlinear and the linear equilibrium paths [1]. The natural frequencies of circular and annular thick FG plates composed of two piezoelectric layers is firstly studied by 3-D Ritz method based on the linear, small strain, and 3-D elasticity theory [2]. As the first elasticity solution for a two-dimensional FGM under applied loading, FG strips and beams subjected to simple tension or bending moment is theoretically addressed on basis of Airy stress function approach [3]. As for advanced curved two-directional FGBs with graded material properties along the axis and thickness directions simultaneously, an analytical model based on the Euler–Bernoulli theory and classical hairbrush governing equations, is proposed for the flexure prediction [4]. However, the above analytical method is limited for linear solution but difficult to solve the geometrically nonlinear behaviors of FGMs, which necessitates numerical methods like finite element method. Considering warping effects, a continuum mechanics based beam element is formulated to analyze the geometrically nonlinear responses of 3D FGB under pure bending, pure torsion, lateral buckling load and coupled stretching-bending-twisting load [5]. The gradual variation of Poisson's ratio is rarely considered but can be particularly important for correctly reproducing the strain and stress fields along the beam thickness, resulting in an investigation of its influences on the displacements, strains and stress based on 2D beam element [6].

To avoid mesh-related problems, meshless methods have drawn much attention and applied in FGMs. A meshless local natural neighbour interpolation (MLNNI) method is adopted to study quasi-static and transient dynamic responses of 2D viscoelastic FG structures [7]. The collocation method based on multi-quadric radial basis functions (RBF) is applied to investigate the static response of FGB incorporating hierarchical beam theories [8]. The optimized volume fraction was obtained to maximize the first natural frequency of FGB using a meshless RBF numerical method [9]. Bending analysis of sandwich FG cylinders is performed by asymptotic meshless method using the differential reproducing kernel interpolation and perturbation method [10], where the effective material properties are determined using the Mori–Tanaka

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scheme. Subjected to combined thermal and mechanical loads, thermo-elastoplastic analysis of thick FG plates is realized by employing a meshless local Petrov-Galerkin (MLPG) method [11].

In contrast to FGB with uniform thickness, FGB with variable thickness is attracting the interests of researchers in the last several years, due to structural matching, strength dispersion, structural weight lightening and economization. On the basis of two-dimensional elasticity theory and Fourier sinusoidal series expansions, bending analysis is firstly implemented for FGB with linearly and quadratically varying thickness [12]. Sari et al. [13] studied the frequency and mode veering phenomena of axially FG tapered beam based on Timoshenko theory. The Euler-Bernoulli beam theory is also used to analyze the free vibration of FGB with variable cross-section on an elastic foundation and spring supports [14]. The free and forced vibrations of axially Timoshenko FGB with variable cross-section on elastic foundation are investigated [15] analytically and numerically by FEM.

To the best of the authors' knowledge, the geometrical nonlinearity has not been considered for FGB with variable thickness and meshless method has not been employed for this type of beams. In this paper, the comparative study of geometrical linear and nonlinear bending deformation of FGB with variable thickness is performed using meshless Smoothed Particle Hydrodynamics (SPH) method. FGM with exponent law distribution is considered and two-dimensional elasticity theory is adopted to construct the stress-strain relationship. The Lagrangian SPH method [16–18] without using mesh connectivity and background cell is attributed to approximate the strain field. The governing equilibrium equation is also discretized and solved by the proposed SPH method and then the displacements are updated using central difference method.

The rest of this paper is outlined as follows: Section 2 details the material properties and governing equations of functionally graded beam with variable thickness. The numerical procedure in SPH method is presented in Section 3. In Section 4, the numerical results are provided and discussed as well as compared to analytical solution in literature and FEM results. Finally the conclusion is drawn in Section 5.

2. Statement of the problem

2.1. Homogenization of the material properties

Considering a Functionally Graded Beam (FGB) with variable thickness shown in Fig. 1, the upper side is horizontal and bears external force. The bottom side is smoothly curved and stress free. The boundary condition taken into account in this paper is simply supported at two ends of the lower side.

The FGB is often constituted by a mixture of metals and ceramics of which the volume fractions are varied continuously through the thickness direction. The most common method to characterize the graded properties in this inhomogeneous material is the continuum model described by the exponent-law or power-law distribution of the components volume fraction. In the present investigation, the material properties of FGB is assumed isotropic at any point and varied smoothly in the thickness according to exponent-law distribution,

$$E(z) = E_0 \exp(\lambda z/H) \tag{1}$$

$$\rho(z) = \rho_0 \exp(\lambda z/H) \tag{2}$$

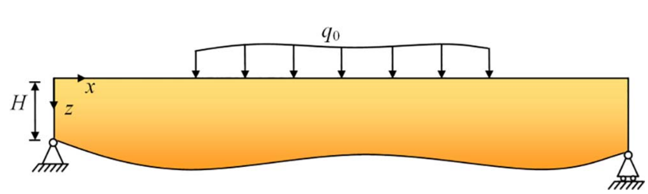

Fig. 1. Functionally graded beam with variable thickness.

$$\nu(z) = \nu \tag{3}$$

where E, ν and ρ are Young's modulus, Poisson's ratio and mass density, respectively. The subscript '0′ means the value at the top surface (z = 0). The parameter λ is gradient index to characterize the material enrichment extent along the thickness direction. The beam is full metallic when λ = 0, and conversely full ceramic when λ = ∞.

2.2. Governing equations

Without loss of generality, the mechanical formulations for general 2D linear elastic solid is provided for solving the deformation of this isotropic inhomogeneous FGM. The elastodynamic equilibrium equation over a domain Ω bounded by a surface Γ can be written

$$\sigma_{ij,j}(\mathbf{x},t) + b_i(\mathbf{x},t) = \rho(z)\ddot{u}_i(\mathbf{x},t) + c\dot{u}_i(\mathbf{x},t) \ (\text{in } \Omega,\ i,j = 1,2) \tag{4}$$

where σ_{ij} is stress tensor, b_i is body force vector. u_i is displacement vectors and c is the damping coefficient. It should be mentioned herein that a subscript after a comma denotes partial derivative with respect the coordinate related to this subscript. Time derivatives are expressed by a dot (first derivative) or two dots (second derivative). The conventional Einstein's summation rule over repeated indices is applied throughout the paper.

The boundary conditions can be expressed by

$$u_i(\mathbf{x},t) = \bar{u}_i(\mathbf{x},t) \quad (\text{on } \Gamma_u) \tag{5}$$

$$t_i(\mathbf{x},t) = \sigma_{ij}(\mathbf{x},t)n_j(\mathbf{x}) = \bar{t}_i(\mathbf{x},t) \quad (\text{on } \Gamma_t) \tag{6}$$

where $\bar{u}_i$ and $\bar{t}_i$ are the prescribed displacement on the essential boundary Γ_u and traction on the natural boundary Γ_t. t_i and n_j signify the surface traction vector and the unit vector outward normal to the boundary Γ ($\Gamma = \Gamma_t \cup \Gamma_u$).

For an isotropic elastic material, the most general relation between the stress and strain can be stated as

for plane stress assumption:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{Bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{Bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{7}$$

for plane strain assumption:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{Bmatrix} 1-\nu & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2} \end{Bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{8}$$

The kinematic relations for the strain field is related to the corresponding displacement

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{i,k}u_{j,k}) \tag{9}$$

When the displacements are small enough that the third term in the equation can be negligible, it is considered as geometrically linear problem.

3. Implementation of SPH method

3.1. Brief introduction of SPH method

Smoothed Particle hydrodynamics (SPH) method was invented for solving problem in astrophysics [16,17,19–21] and then widely used in the fluid dynamics [22–24]. The truly meshless feature without using neither mesh nor background cell, and Lagrangian character to trace the movement history and crack growth path makes it quite attractive in the field of solid mechanics [25–27]. This also promotes the application of SPH method for simulating the bending deformation of FGB.

The continuous domain is discretized with a finite number of unconnected particle scattered in space and each SPH particle possesses

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