Composite Structures 186 (2018) 315–323

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Vibration analysis of multiple-stepped functionally graded beams with general boundary conditions

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ARTICLE INFO

Keywords:
Variational method
Free vibration
Stepped beams
Functionally graded materials

ABSTRACT

In this paper, an effective formulation for vibration analysis of multiple-stepped functionally graded beams with general boundary conditions is presented. The material properties are assumed to change continuously in the thickness direction according to a power law distribution of the volume fraction of the constituents. The theoretical model is formulated on the basis of a variational method in conjunction with the first-order shear deformation theory. The essence of the present formulation is to express the displacement and rotation components by nodeless Fourier sine functions and nodal Lagrangian polynomials. Since the boundary nodal displacement information is introduced into the admissible functions, the interface continuity and boundary conditions are easily handled. Based on this, each structure component may be further partitioned into appropriate segments in order to accommodate the computing requirements of higher-order vibration modes. A variety of numerical examples are presented to demonstrate the accuracy, reliability and computational efficiency of this method. Furthermore, the effects of the material properties, geometric parameters as well as boundary conditions on the frequencies of the beam structures are discussed.

1. Introduction

Functionally graded materials (FGMs) are classified as novel composite materials which possess continuous and smooth spatial variations of material properties along desired directions. Such materials can eliminate the high stress concentration in conventional laminated composite structures. Consequently, FGMs have more extensive potential applications in various engineering fields. As one of the most fundamental structure elements, beam structures made of FGMs need to be well-designed due to there may exists excessive vibration in their applications.

A close scrutiny of the references reveals that a lot of research efforts have been devoted to free vibration analysis of uniform FGM beams. Aydogdu and Taskin [1] analyzed the free vibration of functionally graded beams with simply supported edges using Navier method based on different beam theories. Li [2] proposed a unified approach for analyzing static and dynamic behaviors of FGM beams with the rotary inertia and shear deformation included, in which a single fourth-order partial differential equation were derived by introducing a new auxiliary function. Free and forced vibration analyses of a FGM Euler-Bernoulli beam were carried out by Şimşek and Kocatürk [3] using Lagrange's equations combined with Lagrange multipliers. Subsequently, Şimşek [4] employed different higher-order beam theories to study fundamental frequency characteristics of FGM beams. Sina et al. [5] analyzed free vibration of FGM beams based on a new first-order shear deformation beam theory. Giunta et al. [6] investigated free vibration of a simply supported FGM beam using hierarchical beam theories, in which the three-dimensional kinematic field is derived in a compact form as a generic *N*-order polynomial approximation. Thai and Vo [7] developed various higher-order shear deformation beam theories for bending and free vibration of FGM beams in which the transverse displacement was partitioned into bending and shear components. Pradhan and Chakraverty [8] employed Rayleigh-Ritz method to analyze free vibration of Euler and Timoshenko FGM beams with various boundary conditions. Li et al. [9] studied transverse vibration of axially FGM beams with various end conditions. Free vibration of Bernoulli-Euler FGM beams was analyzed by Su et al. [10] using dynamic stiffness method.

Due to their capability of accommodating more complicated engineering requirements stepped beams are extensively used. However, discontinuous variation in cross-section results in mathematical and computational complexities. There exist a number of reports available

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https://doi.org/10.1016/j.compstruct.2017.12.018
Received 26 April 2017; Received in revised form 27 October 2017; Accepted 11 December 2017
Available online 12 December 2017
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on the vibration analysis of stepped beams. However, most of them are confined to isotropic stepped beams. Naguleswaran [11] proposed an analytical method to study vibration of an Euler-Bernoulli beam with up to three step changes in cross-section, in which the classical and elastic end supports are considered. Koplow et al. [12] obtained the dynamic response of Euler-Bernoulli beams with one step change subjected to free boundary conditions. The free vibration of a cantilevered multiple stepped beam was analyzed by Jaworski and Dowell [13] in which comparison of several numerical methods with experiment was presented. Lu et al. [14] investigated dynamic behavior of a Euler-Bernoulli beam with multiple steps via a composite element method (CEM) which is a combination of the conventional finite element method and highly precise classical theory. Miao [15] extended adomian decomposition method (ADM) to obtain the natural frequencies of multiple stepped beams. Wang and Wang [16] carried out free vibration analysis of Euler-Bernoulli multiple stepped beams via the differential quadrature element method (DQEM). Duan and Wang [17] obtained accurate high-order mode frequencies of Euler-Bernoulli multiple stepped beams using the discrete singular convolution (DSC). Lee [18] utilized a Chebyshev-tau method to analyze free vibration of stepped beams on the basis of the Euler-Bernoulli beam theory and the Timoshenko beam theory. Compared to the isotropic stepped beam, the investigation of FGM stepped beams is rare. Up to now, to the authors' best knowledge, there are only two available papers concerning this problem [19,20] which are limited to FGM beams with one step change and neglect the effects of shear deformation and rotary inertia. Consequently, the primary object of this paper is to establish a reliable and efficient model for vibration analysis of stepped functionally graded beams which takes effects of shear deformation and rotary inertia into account and is capable of handling multiple step changes.

In this paper, an effective formulation is proposed to analyze dynamic characteristic of multiple-stepped Timoshenko beams with general boundary conditions. The material properties are assumed to change continuously in the thickness direction according to a power law distribution of the volume fraction of the constituents. The theoretical model is formulated on the basis of a variational method in conjunction with the first-order shear deformation theory. The essence of the present formulation is to express the displacement and rotation components by nodeless Fourier sine functions and nodal Lagrangian polynomials. Since boundary nodal displacement information is introduced into the admissible functions, the interface continuity and boundary conditions are easily handled. Based on this, each structure component may be further partitioned into appropriate segments in order to accommodate the computing requirements of higher-order vibration modes. A variety of numerical examples are presented to demonstrate the accuracy, reliability and computational efficiency of this method. Furthermore, the effects of the material properties, geometric parameters as well as boundary conditions on the frequencies of the beam structures are discussed.

2. Theoretical formulations

2.1. Description of model

The geometry of a stepped beam consisting of N uniform sections with an aligned neutral axis and the co-ordinate systems related to each section are depicted in Fig. 1. The reference surface of the beam is taken to be its middle surface where the co-ordinate system is fixed. The length, thickness and width of the nth section are denoted by L_n, h_n and b_n. In order to deal with the discontinuous variation in cross-section, the stepped beam is divided into N sections along the location of steps. Each section can be further decomposed into J segments for the purpose of accommodating requirements of high-order vibration modes. The dotted lines in Fig. 1 represent the interface between two adjacent beam segments in a beam section. $\overline{u}_{n,j}$ and $\overline{w}_{n,j}$ are the displacements of the jth segment of the nth section in x and z directions, respectively.

For the typical FGMs fabricated from a mixture of two material constituents M_1 and M_2, material properties are assumed to vary continuously and smoothly in thickness direction. According to Voigt model, the effective Young's modulus E_f, mass density ρ_f and Poisson's ratio μ_f of the nth beam section are defined as

$$E_{n,f} = (E_1-E_2)V_{n,1} + E_2, \quad \rho_{n,f} = (\rho_1-\rho_2)V_{n,1} + \rho_2, \quad \mu_{n,f} = (\mu_1-\mu_2)V_{n,1} + \mu_2 \tag{1}$$

where $V_{n,1}$ is the volume fraction of M_1 in the nth beam section and given as according to a power law distribution

$$V_{n,1} = \left(\frac{z}{h_n} + \frac{1}{2}\right)^{p_n} \qquad n = 1,2,\cdots,N \tag{2}$$

in which p_n is the gradient index which determines the material variation profile through the thickness. Obviously, the top and bottom surfaces of each beam sections are M_1 rich and M_2 rich, respectively. In order to clearly demonstrate the material distribution at stepped points, the schematic sketches of two adjacent beam sections are depicted in Fig. 2. Since the thicknesses of the adjacent beam sections are different, the materials at p_n are mixture of M_1 and M_2, whereas the materials at p_{n+1} are M_1.

2.2. Variational formulation for beam segments

Mathematically, structural vibration problems can be stated in a variational form, which usually leads to a solution that is easier to obtain than by solving the partial differential equations directly. Consequently, in this work, an efficient variational method is implemented to deal with the beam segments. The variational energy of the jth segment in the nth section is defined as

$$\Pi_{n,j} = U_{n,j}-T_{n,j} \tag{3}$$

where $U_{n,j}$ and $T_{n,j}$ are the linear elastic strain and kinetic energies, respectively.

Based on the first-order shear deformation theory, the displacements of an arbitrary point in interior of the beam segment are expressed in terms of displacement and rotation components of the reference surface, as given below:

$$\overline{u}_{n,j}(x,z,t) = u_{n,j}(x,t) + z\varphi_{n,j}(x,t), \quad \overline{w}_{n,j}(x,z,t) = w_{n,j}(x,t) \tag{4}$$

where $u_{n,j}$ and $w_{n,j}$ are the displacement components of corresponding point on middle surface in x and z directions, respectively. $\varphi_{n,j}$ is the rotation of transverse normal. For the sake of simplicity, the subscripts n and j will be omitted in the following formulation. t is time variable.

The linear strain–displacement relations in the interior of the domain can be given as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & z\frac{\partial}{\partial x} & 0 \\ 0 & 1 & \frac{\partial}{\partial x} \end{bmatrix} \begin{Bmatrix} u \\ \varphi \\ w \end{Bmatrix} = \begin{bmatrix} 1 & z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \chi \\ \gamma \end{Bmatrix} \tag{5}$$

where ε_0 is the membrane strain of mid-surface. χ is the curvature change. γ is the transverse shear strain. The constitutive relations are given in the matrix form:

$$\begin{Bmatrix} \sigma_{xx} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \gamma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & zQ_{11} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_0 \\ \chi \\ \gamma \end{Bmatrix} \tag{6}$$

where σ_{xx} and τ_{xz} are the normal and shear stresses, respectively. The elastic coefficients Q_{11} and Q_{66} are functions of variable z and defined as

$$Q_{11} = E_f, \quad Q_{66} = \frac{E_f}{2[1 + \mu_f^2]} \tag{7}$$

The relationships between force moment resultants and strains are defined by:

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