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Lateral torsional buckling of anisotropic laminated thin-walled simply supported beams subjected to mid-span concentrated load

Habiburrahman Ahmadi, Hayder A. Rasheed*

Department of Civil Engineering, Kansas State University, Manhattan, KS 66506, United States

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ABSTRACT

In this paper, a generalized semi-analytical approach for lateral-torsional buckling of simply supported anisotropic, thin-walled, rectangular cross-section beams under concentrated load at mid-span/mid-height was developed using the classical laminated plate theory as a basis for the constitutive equations. A closed form buckling expression was derived in terms of the lateral, torsional and coupling stiffness coefficients of the overall composite. These coefficients were obtained through dimensional reduction by static condensation of the general 6x6 constitutive matrix mapped into an effective 2×2 coupled weak axis bending-twisting relationship. The resulting two coupled stability differential equations are manipulated to yield a single governing differential equation in terms of the twisting angle. This differential equation with variable coefficients, subjected to applicable boundary conditions, was solved numerically using Mathematica. The resulting solution was found to correlate with the effective lateral-flexure, torsional and coupling stiffness coefficients to yield a general semi-analytical solution. An analytical formula was possible to extract, which was verified against finite element buckling solutions using ABAQUS for a wide range of lamination orientations showing excellent accuracy. The stability of the beam under different geometric and material parameters, like length/height ratio, layer thickness, and ply orientation, was investigated.

1. Introduction

Thin-walled beam structures are major components in many engineering applications. They are widely used as structural components in many types of systems in the field of aerospace, mechanical, marine, and civil engineering. Fiber reinforced polymer (FRP) composites, are replacing conventional materials in some of these types of structural systems. This increase in interest for using FRP lies in some critical advantages of these materials over conventional counterparts. Their high strength to weight ratio, high stiffness to weight ratio, their corrosion resistance, their ease of transportation and erection, and their fatigue resistance are some of the advantages FRP offer. The most distinguished characteristic is the ability of tailoring the material for each particular application. Structural properties depend on the material system and the shape of the cross-section of the member [3]. Unlike isotropic shapes, composite members are possible to optimize by altering the material itself through choosing among a variety of resins, fiber systems, and fiber orientations. Although thin-walled FRP structures exhibit high strength, problems of excessive deformation and instability, due to their low stiffness and slenderness of the member, are the major disadvantages in wider acceptance for structural engineering applications [15]. Because of these limitations, the new generation of composite structures should be designed to work in a safe way and to experience higher performance than the conventional systems. Consideration of stability and deformation limit states tends to be the governing design criteria for FRP structures before these structures reach material failure. Thus, the proper establishment of such criteria is an important prerequisite to the practical use of FRP in engineering applications.

A thin-walled slender beam subjected to in-plane bending moments (about the strong axis) may buckle in a combined lateral bending and twisting of the cross-section. This phenomenon is known as lateral-torsional buckling. Theory of thin-walled open section beams including axial constrains for isotropic materials was developed by Vlassov [24]. This classical theory neglects shear deformation in the middle surface of the wall so for thicker-walled beams, the shear deformations may significantly increase the displacements and reduce the buckling loads. The shear deformation theory for transversely loaded isotropic beams was developed by Timoshenko and Gere [23].

Many researchers have focused on studying the lateral torsional buckling of composite beams using different theoretical approaches while validating their work with experimental programs or finite

* Corresponding author.
E-mail address: hayder@ksu.edu (H.A. Rasheed).

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element analysis. Among those, Bauld and Lih-Shyng [5] applied Vlasov's theory for open section composites with symmetrical laminated walls neglecting the shear deformation. Bank and Bednarczyk [2] and Barbero et al. [3] developed simple expressions for the bending, torsional, and warping stiffness of composite laminated beams. Sherbourne and Kabir [21] studied analytically the effect of transverse shear strain on the lateral buckling of thin-walled, open-section fibrous composite beams. Pandey et al. [17] proposed an analytical formulation for finding the optimal direction of fiber for improving the lateral buckling strength of thin-walled I-section composite beams. Lin et al. [15] studied stability of thin-walled composite structural members using finite element method. Davalos and Qiao [6] used the non-linear elastic theory to develop a stability solution for lateral-distortional buckling for wide flange composite beams based on the principle of total potential energy. Kollar [10] suggested a closed form solution for thin-walled open section columns, made of orthotropic composite materials, by considering flexure, shear and the torsional warping induced shear deformations. Roberts and Al-Ubaidi [19] studied the influence of shear deformation on restrained torsional warping of pultruded FRP bars of open cross-section by proposing an approximate theory. Sapkás and Kollár [20] studied the stability analysis of thin-walled, open section beams, made of orthotropic composite materials under various loading conditions. The solution indirectly accounted for shear deformation by adjusting the bending and warping stiffness of the composite beams. Lee et al. [13] presented a general analytical model, based on the classical lamination theory, applicable to the lateral buckling of composite laminated I-beams subjected to various types of loadings. Qiao et al. [18] studied a combined analytical and experimental evaluation of flexural-torsional buckling of fiber reinforced polymer composite I-beams based on energy method developed from the non-linear plate theory. Tai [22] studied lateral- torsional buckling of symmetrically laminated, rectangular cross-section, composite beams under various loading conditions. Kotelko et al. [12] studied theoretical analysis for local buckling of different cross section thin-walled beams and columns. Yoon et al. [26] derived explicit numerical expressions to predict the complex dynamic behavior of thin-walled curved beams by formulating stiffness and mass matrix of the curved beam element for finite element analyses. Additional warping degree of freedom at each node was taken into account. Yau [25] presented a finite element model for buckling analysis of tapered I-beams subjected to torsional moments by representing the torsional moments as the combination of St. Venant and warping torsions. He indicated that the warping and St. Venant torsions may significantly affect the flexural buckling resistance of tapered I-beams against torsional loadings. Furthermore, the critical loads of the tapered I-beams with higher warping rigidity would be reduced by the instability effect of warping moments.

Karaagac et al. [9] studied stability of cantilever laminated symmetric and anti-symmetric composite beams under static and dynamic conditions by applying elastic support. Goyal and Kapania [7] proposed a 21 degree-of-freedom beam element, based on the first-order shear deformation theory, to study the response of unsymmetrically laminated composite structures subject to both static and dynamic problems. They employed an accurate model to obtain the transverse shear correction factor. Machado [16] studied the stability of simply supported thin-walled symmetric laminated composite I-beams subjected to combined axial and lateral loads by approximate analytical solutions and compared them with numerical results. The proposed solution also examined the nonlinear pre-buckling geometrical deformation for more accurate representation of the lateral stability conditions. He and Yang [8] modified Reddy's higher order beam theory to ensure the satisfaction of interfacial shear force continuity condition of two-layer composite beams, and the static and buckling analyses were carried out using finite element analysis.

Most of the work, related to lateral- torsional stability of thin-walled composite beams, was focused on I-sections. The beams were either considered to be of symmetric layup, anti-symmetric layup, orthotropic, or pultruded in nature. There has not been any study recorded on the behavior of general anisotropic laminated composite beams to the best knowledge of the authors.

In the present study, a generalized semi-analytical model applicable to the lateral-torsional buckling of a simply supported rectangular cross-section beams, made of anisotropic laminated composite materials, subjected to concentrated load at mid-span/mid-height is developed. This model is based on the classical laminated plate theory (CLPT), and accounts for the arbitrary laminate stacking sequence configurations. A finite element model is developed in ABAQUS to predict critical buckling loads and compare them with the results obtained from the analytical model. The effects of fiber orientation, beam length/height ratios and wall thickness on the critical buckling forces are studied. A finite element nonlinear pre-buckling geometrical deformation analysis was also examined and compared against the semi-analytical solution to examine the existence of pre-buckling deformation.

2. Analytical formulation

A simply supported laminated composite beam with length L and a thin rectangular cross section is subjected to concentrated load at mid-span/mid-height, as shown in Fig. 1. The model in this study is based on the classical laminated plate theory, Barbero [4], Kollár and Springer [11], and Lee et al. [14]. The following assumptions are adopted from the classical laminated plate theory:

1. The normal to mid-plane (reference surface) of the laminate remain normal and straight after deformation.
2. The normal to mid-plane of the laminate does not change length (i.e. the thickness of the laminate stays constant).
3. The shear deformations are neglected.
4. The laminate consists of perfectly bonded layers.
5. The stress-strain relationships are applied under plane-stress conditions.

The beam tends to buckle under a lateral-torsional behavior because of its small thickness. The beam is divided into two halves from the point of applied load (mid-span) in order to derive the buckling equation. The coordinate system is assigned from the support end of each half as x-axis to be positive moving towards the center of the beam. The angle of twist, β, is positive (counterclockwise buckling) in the left side of the beam (Fig. 1a), while it is negative (clockwise buckling) in the right side of the beam (Fig. 1b). The buckling equation is derived for each side of the beam, separately. Then, the resulting equations are reconciled together. Fig. 1c shows the boundary conditions for the beam shown from top view.

2.1. Kinematics

Based on the assumptions of the classical laminated plate theory, the displacement components u, v, w representing the deformation of a point in the plate profile section are written in terms of the mid-surface displacements u_0, v_0, and w_0 as well as the two rotations $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y} = \beta$, see Fig. 1, [4]. By differentiating the displacement expressions of u and v with respect to x, y as well as x and y, the strains in the plate are defined in terms of the mid-surface strains and curvatures:

$$\varepsilon_x = \varepsilon_x^0 + z\kappa_x \tag{1}$$

$$\varepsilon_y = \varepsilon_y^0 + z\kappa_y \tag{2}$$

$$\gamma_{xy} = \gamma_{xy}^0 + z\kappa_{xy} \tag{3}$$

where