Composite Structures 185 (2018) 393–400

Contents lists available at ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

Post-buckling behavior and nonlinear vibration analysis of a fluid-conveying pipe composed of functionally graded material

Ye Tang[a], Tianzhi Yang[b,*]

[a] School of Mechanical and Automotive Engineering, Anhui Polytechnic University, Wuhu 241000, PR China
[b] Department of Mechanics, Tianjin University, Tianjin 300072, PR China

ARTICLE INFO

Keywords:
Fluid-conveying pipe
Functionally graded material
Post-buckling behavior
Nonlinear vibration

ABSTRACT

The post-buckling behavior and nonlinear vibration of a fluid-conveying pipe composed of a functionally graded material were analytically studied. The power-law material property was considered as continuously varying across the direction of the pipe wall thickness. A nonlinear governing equation for the pipe and relevant boundary conditions were derived based on Hamilton's principle. The post-bucking configurations of the pipe were analytically predicted. The closed-form expression of the nonlinear free vibration of the pipe was determined using the homotopy analysis method. Numerical results are presented to display the dependence of the flow velocity, fluid density, and the initial stress on the post-buckling configurations. It was concluded that the statics and dynamics are significantly changed by the material properties, which suggests that the dynamic behavior of pipes may be tailored by use of man-made functionally graded materials.

1. Introduction

The dynamics of fluid-conveying pipes induced by the solid–fluid coupling is a significant scientific issue that involves numerous practical applications in many industrial fields, particularly in power transmission and the nuclear, chemical, and oil industries. Excessive vibration and noise may limit their applications; therefore, this topic has received extensive attention and is still of interest today [1–6].

The earliest recent investigations [7] studied linear free vibrations, including piping stability and natural frequencies, for various boundary conditions. Analytical and numerical techniques, such as the homotopy method [8], the spectral element algorithm [9], the wave approach [10], and the finite-element method [11], were utilized to analyze the dynamic behavior of pipe–fluid interactions. Some classical phenomena were discovered, including fluttering and buckling [12]. Moreover, some new physical phenomena in complicated pipes were explored; for example, An and Su [13] employed the generalized integral transform technique to investigate dynamic problems of pipes conveying liquid–gas two-phase flow and found that the volumetric qualities have significant impacts on the frequencies and vibration amplitudes of the pipes. Dai and Wang [14] investigated the stability of fluid-conveying pipes constituting by two different materials and indicated that the piping system would display instability–restabilization–instability behaviors for high flow velocities. Sadeghi and Karimi-Dona [15] used finite-element and state-space algorithms to analyze the dynamics of a fluid-conveying pipe subjected to a moving sprung mass. They found that a higher moving mass velocity led to a lower natural frequency. Wang et al. [16–17] explored the mode change and unstable modes of cantilevered and conical pipes using theoretical and experimental methods.

Parametric resonance induced by pulsatile flow is another important issue. Yang et al. [18] applied the multi-scale method to discuss the stability of a simply supported pipe conveying pulsatile flow in subharmonic and combination resonances. McDonald and Namachchivaya [19] used the global bifurcation method to investigate parametric vibrations of a supported pipe conveying pulsatile fluid in the vicinity of 0:1 resonance. Panda and Kar [20] numerically examined the stability, bifurcation and vibration response of a hinged pipe undergoing a combination of parametric and internal resonances. Seo et al. [21] proposed a finite-element formulation to study the stability and forced response of fluid-conveying pipes with pulsatile flow. Zhai et al. [22] derived the motion equation for Timoshenko curved pipes conveying fluid using Hamilton's principle and employed the pseudo-excitation method to discuss dynamic responses under the condition of random excitation. Qian and Wang [23] used the differential quadrature method to study the stability of a pinned pipe under thermal loads. Chang and Modarres-Sadeghi [24] utilized Galerkin and finite-difference methods to examine the three-dimensional dynamics of a cantilevered pipe subjected to a base excitation. Jin [25] applied numerical simulations to investigate the stability and chaos dynamics of a

* Corresponding author.
E-mail address: yang@dyn.tu-darmstadt.edu (T. Yang).

https://doi.org/10.1016/j.compstruct.2017.11.032
Received 21 August 2017; Received in revised form 4 November 2017; Accepted 13 November 2017
Available online 21 November 2017

cantilevered pipe in the case of motion-limiting constraints. To avoid failure caused by large displacements of pipes conveying fluid, some researchers adopted the optimized parameters' technique [26] and the active control method [27] to suppress vibration of pipes. Yang et al. [28] proposed a passive and adaptive method to stabilize fluid-conveying pipes based on non-linear targeted energy-transfer theory and, from numerical results, the method was proved to be effective.

The materials properties of the pipes in all of the above-mentioned studies were all isotropic and homogeneous. To clearly reveal the physical parameters and dynamic behavior of a functionally graded material (FGM) pipe, we used the homotopy analysis method to find a closed-form solution to the nonlinear free vibration problem.

FGM is a composite material comprising two or more different materials [29]. It can be designed as an inhomogeneous material with its structure varying continuously along a preferred orientation, in which the distribution of its physical properties is graded according to the volume fraction law. Compared with conventional composite materials, such as laminate, a notable distinction is that FGMs can diminish stress concentration [30,31], increase bond strength [32], improve fracture toughness [33], and enhance corrosion resistance [34]. Furthermore, the desired mechanical characteristics of an FGM, including material density, Poisson's ratio, and Young's modulus, can be tailored by adjusting variations of the volume fraction along the preferred direction [35,36]. FGM has been extensively applied in various engineering structures, such as beam [37–39], plate [40,41], shell [42,43], and other micro- or nano-sized structures [44–47]. FGM has recently been introduced to enhance the stability of piping systems, but relatively few applications are reported. Deng et al. [48] analyzed the instability of a multi-span viscoelastic pipe made of FGM by utilizing the wave-propagation approach and reverberation-ray matrix algorithm. Wang and Liu [49] applied a symplectic approach to study the eigenvalue problem of a clamped FGM pipe and presented numerical results to illustrate the effects of physical parameters, such as the power-law exponent, on its transverse vibration and stability. An and Su [50] employed the generalized integral transform method to analyze the linear vibration frequencies and amplitudes of a functionally graded pipe conveying fluid.

The above-mentioned works, however, only focused on the linear dynamics of FGM pipes conveying fluid. To best of our knowledge, there are no reports on the non-linear vibrations of FGM pipes, which is equally important for pipe research. This work therefore aimed to study the nonlinear free vibration of an FGM pipe conveying fluid and explored interesting phenomena in FGM geometric nonlinear interactions.

2. Equation of motion

Consider a uniform, fluid-conveying FGM pipe, with length L and mean radius r, with the fluid traveling at a flow velocity Γ between two simple supports at both ends, as seen in Fig. 1. The symbols u and w represent the pipe displacement in the x and z directions, respectively. The inner surface $r = r_i$ and the outer surface $r = r_o$ are constituted by

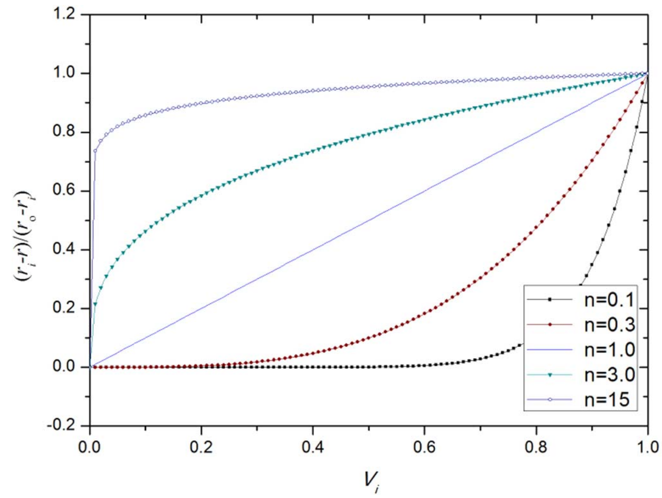

Fig. 2. Variations of metal volume fraction along pipe thickness for different values of power-law exponent n.

different materials. In this work, the material properties of the FGM pipe were assumed to follow a power-law distribution:

$$E(r) = V_i E_i + V_o E_o \tag{1}$$

$$\rho(r) = V_i \rho_i + V_o \rho_o \tag{2}$$

where E and ρ are the elastic modulus and density of the FGM pipe, respectively. Subscripts o and i denote the outer and inner layers, respectively. The volume fraction of the material can be introduced as:

$$V_i = \left(\frac{r_o - r}{r_o - r_i}\right)^n \tag{3}$$

$$V_o = 1 - V_i \tag{4}$$

in which n represents the power-law exponent. It is obvious that the FGM pipe becomes a homogeneous pipe in case of the exponent $n = 0$. The variation of the metal volume fraction across the pipe wall thickness is shown in Fig. 2 for different power-law exponents. The materials properties of FGM pipe depend strongly on the exponent n.

According to the Euler–Bernoulli theory, the longitudinal strain related to displacements of the pipe can be written as:

$$\varepsilon(x) = -z\frac{\partial^2 w}{\partial x^2} + \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{5}$$

where z is the coordinate measured from the plane of the neutral fibers. Assuming that the pipeline is elastic, the stress–strain relation is given by:

$$\sigma = E(r)\varepsilon(x) \tag{6}$$

The strain energy U of FGM pipe comprises two parts: the first part U_1 represents the strain energy originating from a variation of stress

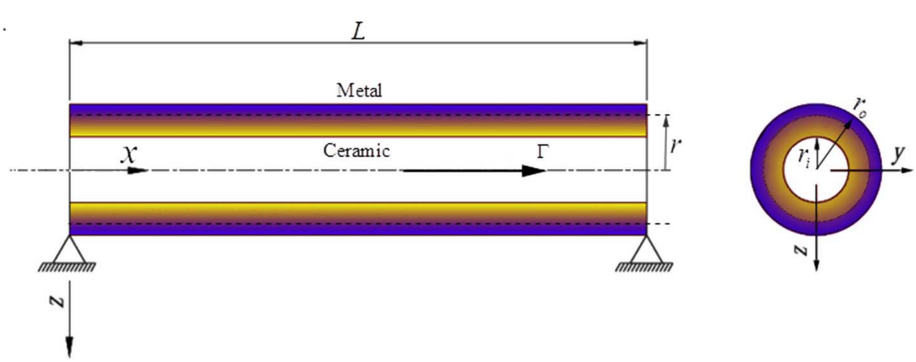

Fig. 1. Physical model of fluid-conveying FGM pipe with simple support at both ends.