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Natural characteristics analysis of magneto-electro-elastic multilayered plate using analytical and finite element method

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A B S T R A C T

For the magneto-electro-elastic multilayered plate, the natural characteristics of regular simply supported three-layered plate is investigated through analytical and finite element method respectively. First, the analytical model of simply supported magneto-electro-elastic multilayered plate is built according to state vector, and then the analytical solution of natural characteristics is obtained by propagator matrix method. Afterwards, the numerical solution of natural characteristics is calculated by finite element method based on the theory of magneto-electro-elastic multilayered plate. Finally, the comparison analysis of analytical and numerical solutions are conducted respectivly, and the results indicate that they have an excellent agreement.

1. Introduction

The magneto-electro-elastic (MEE) composites, first reported by Boomgaard et al., consist of piezoelectric (PE) and piezomagnetic (PM) phases and thus have ability to convert energy among the magnetic, electric and elastic forms [1,2]. Due to their multifunctional capabilities, the MEE composite have been widely applied in sensors, actuators, memory devices, sonar applications, ultrasonic imaging devices and space structures in recent years. Otherwise, MEE composites exhibit coupling between magnetic, electric and elastic material properties, such as magnetoelectric effect which refers to the coupling between the electric and magnetic fields [3,4]. As for the laminated plate of these composites, the multilayered configuration and this capability of passively coupling bring great difficulty to analyze vibration behavior, and motivate a great number of researchers to investigate their natural characteristics through analytical and numerical methods.

E. Pan [5,6] first investigated the nonlinear vibration of anisotropic MEE multilayered plates, and derived the analytical solution for the vibration and static bending of rectangular multilayered plate with simply supported edges. Li et al. [7] analyzed the influence of material heterogeneity and multi-field coupling for a circular plate of intelligent material with transverse isotropy subjected to the thermal loadings. Combined the thin plate theory with von Karman's nonlinear strains, Shooshtari and Razavi [8–10] investigated the nonlinear vibration of rectangular simply supported MEE plate. Ebrahimi et al. [11] presented a four-variable shear deformation refined plate theory to avoid applying shear correction factors, and investigated the influences of magnetic potential, electric voltage, various boundary conditions, etc. on natural frequencies of MEE functionally graded plate. Taking the electro-magnetic influence into account, Milazzo [12] built a nonlinear large deflection model for simply-supported MEE plate and found that the plate thickness have effect on large deflection and electro-magnetic response. Using the third order in-plane kinematics, Milazzo and Orlando [13] built an equivalent single-layer model for the free vibration analysis of smart laminated plate. Combining the state space approach with the discrete singular convolution algorithm, Xin et al. [14] presented semi-analytical solutions for free vibration of simply supported and multilayered MEE plates. Farajpour et al. [15] derived closed-form solutions of natural frequency for the MEE nanoplates subjected to external electric and magnetic potentials. Considering the virtual mass of water-contacting MEE rectangular plates, Chang [16–18] established a fluid-structure interaction model to analyze the nonlinear vibration characteristics of transversely isotropic MEE rectangular plates in contact with fluid. To study the free vibration of a simply supported MEE plate, Chen et al. [19] derived two separated state equations by introducing proper stress and displacement functions. Bravo-Castillero and et al. [20,21] derived the sixteen sets of Thermo-MEE constitutive relations and energy potentials and applied the asymptotic homogenization method to investigate the global behavior of functionally graded Thermo-MEE multilayers. Considering different lateral surface conditions of the plates, Wu and Liu [22] presented the three-dimensional free vibration solutions of functionally graded MEE multilayered

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plates using a modified Pagano method. Concerning linear uncoupled and coupled phenomena, Gu and He [23] derived closed-form expressions for the effective properties of layered composites. Giordano [24,25] proposed an explicit procedure for the homogenization of laminated MEE materials and determined the effective response of a multilayered structure.

Meanwhile, the finite element (FE) method has been widely and successfully applied in the numerical solution of natural characteristics of MEE multilayered plate. Wang et al. [26] developed a three-dimensional FE formulation for the transient response of a bi-layered multiferroic composite plate. Based on the Golla-Hughes-McTavish method, Kattimani et al. [27] built a three-dimensional finite element model for the functionally graded MEE plates integrated with the patches of the active constrained layer damping treatment and analyzed its active damping performance. Based on an equivalent single-layer model, Alaimo et al. [28] derived an original FE formulation for the analysis of large deflections in MEE multilayered plates. Based on a first order mechanical equivalent single-layer model, Alaimo et al. [29] presented an isoparametric four-node finite element for multilayered MEE plates. Milazzo et al. [30] presented a variable kinematics approach for moderately large deflection analysis of smart MEE multilayered plates. Respectively considering the mechanical pressure and electric loading, Chen et al. [31] investigated on the numerical solution for mechanical, electric and magnetic fields of a 3-layer MEE half-space structure. Applying the scaled boundary FE method, Liu et al. [32,33] solved the components of the MEE field in the in-plane and thickness direction, and then demonstrated its versatility through four types of MEE plates. Considering uniform temperature rise and moisture concentration rise, Vinyas et al. [34] investigated the effect of hygrothermal loads, temperature and moisture dependent material properties on static response of MEE plate. For MEE coupling, Koutsawa [35] first computed the interaction tensors between the multiferroics inhomogeneities.

The aim of this work is to investigate the natural characteristics of multilayered plate made of anisotropic MEE composites through analytical and finite element method. This paper is organized as follows: in Section 2 and 3, the analytical and numerical solution approaches are proposed based on propagator matrix method and the theory of MEE multilayered plate. The natural characteristics of three-layered plates with four different stacking sequences are obtained, and the comparison analysis and discussion are present in Section 4. The conclusions are drawn in Section 5.

2. Analytical solution of natural characteristics of MEE multilayered plate

A MEE multilayered plate which is composed of N layers referred to a Cartesian co-ordinate with the x, y and z coordinates spanning its length L_x, width L_y, and thickness H respectively, is shown in Fig. 1. For a certain layer j, its bottom and top interface in z-direction can be defined as z_{j-1} and z_j respectively, and thus its thickness is obtained as $h_j = z_j - z_{j-1}$, $(j = 1, \cdots, N)$.

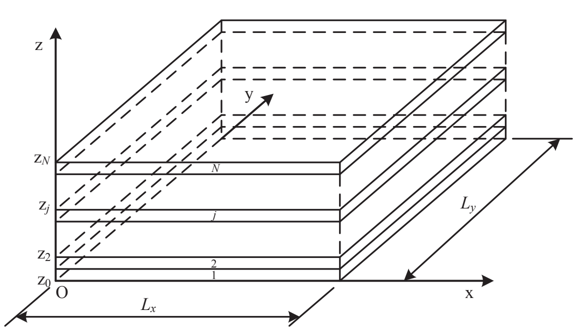

Fig. 1. The MEE multilayered plate.

It is assumed that the extended vectors of displacement and traction are continuous across the layer interface, and the displacement and/or traction on the top and bottom surfaces of the whole plate are also assumed to be zero. The out-of-plane variable vector $\boldsymbol{\eta}1$ and in-plane variable vector $\boldsymbol{\eta}2$ are as follows [4]

$$\eta_1 = [u \quad v \quad D_z \quad B_z \quad \sigma_z \quad \tau_{zx} \quad \tau_{zy} \quad \varphi \quad \psi \quad w]^T \tag{1}$$

$$\eta_2 = [\sigma_x \quad \sigma_y \quad \tau_{xy} \quad D_x \quad D_y \quad B_x \quad B_y]^T \tag{2}$$

The coupled constitutive relation of the MEE multilayered plate can be written as [36]

$$\sigma_i = C_{ik}\gamma_k - e_{ki}E_k - q_{ki}H_k, \quad D_i = e_{ik}\gamma_k + \varepsilon_{ik}E_k + d_{ik}H_k, \quad B_i = q_{ik}\gamma_k + d_{ik}E_k + \mu_{ik}H_k \tag{3}$$

where σ_i, D_i, and B_i are the stress, electric displacement, and magnetic induction (i.e., magnetic flux), respectively; γ_k, E_k, and H_k are the strain, electric field and magnetic field, respectively; C_{ik}, ε_{ik}, and μ_{ik} are the elastic, dielectric, and magnetic permeability coefficients, respectively; e_{ik}, q_{ik}, and d_{ik} are the piezoelectric, piezomagnetic, and magneto-electric coefficients respectively. The coefficient matrices in Eq. (3) are as follows

$$\boldsymbol{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & sym & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix}, \quad \boldsymbol{e} = \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & e_{33} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{q} = \begin{bmatrix} 0 & 0 & q_{31} \\ 0 & 0 & q_{32} \\ 0 & 0 & q_{33} \\ 0 & q_{24} & 0 \\ q_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}, \quad \boldsymbol{d} = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix}, \quad \mu = \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{22} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix}.$$

The general strain-displacement relation, using tensor symbol for the elastic strain γ_{ij}, is expressed as

$$\gamma_{ij} = 0.5(u_{i,j} + u_{j,i}), \quad E_i = -\varphi_{,i}, \quad H_i = -\psi_{,i} \tag{4}$$

where u, ϕ, and ψ are the elastic displacement, electric potential, and magnetic potential respectively.

Substituting Eq. (4) and coefficient matrices into Eq. (3) then yields

$$\sigma_x = c_{11}\frac{\partial u}{\partial x} + c_{12}\frac{\partial v}{\partial y} + c_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \varphi}{\partial z} + q_{31}\frac{\partial \psi}{\partial z}$$
$$\sigma_y = c_{12}\frac{\partial u}{\partial x} + c_{22}\frac{\partial v}{\partial y} + c_{23}\frac{\partial w}{\partial z} + e_{32}\frac{\partial \varphi}{\partial z} + q_{32}\frac{\partial \psi}{\partial z}$$
$$\sigma_z = c_{13}\frac{\partial u}{\partial x} + c_{23}\frac{\partial v}{\partial y} + c_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \varphi}{\partial z} + q_{33}\frac{\partial \psi}{\partial z}$$
$$\tau_{zy} = c_{44}\frac{\partial w}{\partial y} + c_{44}\frac{\partial v}{\partial z} + e_{24}\frac{\partial \varphi}{\partial y} + q_{24}\frac{\partial \psi}{\partial y}$$
$$\tau_{zx} = c_{55}\frac{\partial w}{\partial x} + c_{55}\frac{\partial u}{\partial z} + e_{15}\frac{\partial \varphi}{\partial y} + q_{15}\frac{\partial \psi}{\partial y}$$
$$\tau_{xy} = c_{66}\frac{\partial u}{\partial y} + c_{66}\frac{\partial v}{\partial x}$$
$$D_x = e_{15}\frac{\partial w}{\partial x} + e_{15}\frac{\partial u}{\partial z} - \varepsilon_{11}\frac{\partial \varphi}{\partial x} - d_{11}\frac{\partial \psi}{\partial x}$$
$$D_y = e_{24}\frac{\partial w}{\partial y} + e_{24}\frac{\partial v}{\partial z} - \varepsilon_{22}\frac{\partial \varphi}{\partial y} - d_{22}\frac{\partial \psi}{\partial y}$$
$$D_z = e_{31}\frac{\partial u}{\partial x} + e_{32}\frac{\partial v}{\partial y} + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{33}\frac{\partial \varphi}{\partial z} - d_{33}\frac{\partial \psi}{\partial z}$$
$$B_x = q_{15}\frac{\partial w}{\partial x} + q_{15}\frac{\partial u}{\partial z} - d_{11}\frac{\partial \varphi}{\partial x} - \mu_{11}\frac{\partial \psi}{\partial x}$$
$$B_y = q_{24}\frac{\partial w}{\partial y} + q_{24}\frac{\partial v}{\partial z} - d_{22}\frac{\partial \varphi}{\partial y} - \mu_{22}\frac{\partial \psi}{\partial y}$$
$$B_z = q_{31}\frac{\partial u}{\partial x} + q_{32}\frac{\partial v}{\partial y} + q_{33}\frac{\partial w}{\partial z} - d_{33}\frac{\partial \varphi}{\partial z} - \mu_{33}\frac{\partial \psi}{\partial z}$$