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# Electrically conducting sandwich cylinder with a planar lattice core under prescribed eigenstrain and magnetic field

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ABSTRACT

The eigenstrain theory is widely used to study inelastic responses of several classes of materials subjected to phase transformation, thermal expansion, and other multiphysics excitations. In this paper, we focus on electrically conducting cellular solids and examine their magnetoelastic responses when used as a core of a sandwich cylinder subjected to an eigenstrain and an external magnetic field. The cylinder comprises layers of either solid or cellular material and undergoes either plane strain or plane stress conditions in both time-harmonic and transient states. We use direct homogenization techniques (standard mechanics and micromechanical models) along with Bessel, Struve, and Lommel functions to study the roles that cell topology, relative density, eigenstrain, and bonding interface play on the magnetoelastic responses of the sandwich cylinder. The results show that relative density, cell topology, and magnetic field are the factors that most contribute to control the sandwich response. We also show that a careful tailoring of relative density and cell topology can lead to the simultaneous weight and overall stress reduction with improved natural frequency.



## 1. Introduction

In a homogenous body free from external forces and surface constraints, inclusion refers to a finite subdomain subjected to a prescribed eigenstrain [1]. Thermal expansion, swelling strain, phase transformation, plastic deformation, and piezoelectric/piezomagnetic strain are examples of nonelastic strains, namely eigenstrains [2–4]. The eigenstrain theory enables in a single mathematical formulation to study each of these phenomena, including the response of an electrically conducting material. It can also be used to study the impact of microstructural imperfections in homogeneous and heterogeneous media.

Several works in the literature resorted to the eigenstrain theory to study problems dealing with the three-dimensional nanostrain in Ni–Ti shape memory alloys [5], residual strain measurements [6], stress changes caused by local deboning and damage evolution [7], and strain in living or non-living tissues [8]. Early micromechanical investigations on the eigenstrain were pioneered by Eshelby [9] as well as Mura and Kinoshita [10]. Since then, the studies that followed mainly focused on the application of eigenstrain in particulate composites and residual stress measurements [11–22]. For example, Liang et al. [23] obtained the stress field induced by an eigenstrain within an ellipsoidal inclusion in a thin film of a microelectromechanical system. The effect of the thin film's thickness on the induced eigenstress was found to decrease with an increase in the film thickness. The problem of an arbitrary-shaped heterogeneity, undergoing an eigenfield in a uniform magnetoelectroelastic load, was also examined. Shen and Hung [24], for instance, observed that the selection of appropriate eigenfields could effectively reduce the eigenstress developed in piezoelectric and piezomagnetic composites.

Sandwich structures are routinely used in automotive, aerospace, and sports equipment, among other sectors [25]. In the literature, there exists a large body of research on this subject across the length scale spectrum. In general, previous works focus on the micromechanics and multiphysics responses, such as elastodynamic, thermoelastic, and torsional rigidity [26–28]. The core of a typical sandwich panel is commonly made of either foams, e.g.

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polyurethane and polyethylene foams, or periodic cellular solids, e.g. honeycombs, corrugated metals, and other lattices [29,30]. Their cellular architecture is often preferred to their solid counterpart not only for their low mass, but also for their ability to satisfy multifunctional requirements, such as those prescribed for heat exchangers, piezoelectric transducers, and bone tissue scaffolds [31–33]. For example, thermal expansion, electric conductivity, and flow permeability are some of the properties controlled by the geometry of the unit cell, in particular its topology, nodal connectivity, and relative density. While there are many studies on the multifunctional properties of cellular materials [34–43], few of them have used an eigenstrain analysis to study their mechanical and multiphysics properties, and to investigate the residual stress within the sandwich structure. For example, Schjodt-Thomsen and Pyrz [44] used a model based on Eshelby's equivalent eigenstrain to analyze the influence of alternative statistical cell dispersions on the local strain and the overall effective properties. The model was capable of capturing the effect of microstructural morphology on the stiffness properties. Nguyen et al. [45] examined the homogenization problem by imposing an eigenstrain that represents the thermal or piezoelectric strain within the representative volume element (RVE). The authors highlighted the simplicity of their model which was then used to obtain the effective properties. Moreover, Liu and Liang [46], through a micromechanics approach, obtained the effective elastic moduli of triangular lattices with microstructural defects. In this case, the use of eigenstrain was proved to be effective in the assessment of the role of defects occurring during the manufacturing of a set of cellular solids.

More recently, electrically conducting cellular structures under a prescribed magnetic field have garnered a great deal of attention in a range of applications, such as medical microrobots driven by magnetic actuations [47], hydrophones [48], and metamaterials with negative permeability and negative refraction properties for optics [49]. Advances in additive manufacturing have also enabled the fabrication of cellular solids with electrically conducting microarchitecture, specifically for real-time measurement of structural performance [50–52]. Sandwich structures with electrically conducting solids experience the Lorentz force, according to the uncoupled magnetoelasticity theory [53], which dictates their multifunctional responses. To date, however, the behavior of a sandwich cylinder with an electrically conducting cellular core subjected to a prescribed magnetic field has been rarely studied in the literature. To the best knowledge of authors, the analysis conducted in reference [37] is the only contribution considering the effect of magnetic field on the behavior of porous sandwich structures where the cellular layers have square cells. This subject matter is thus the focus of this work, which contributes to the body of literature dealing with the multiphysics analysis of cellular materials in the presence of eigenstrain.

The following clarifies the differences and originality that distinguish this work from that in reference [37]. This paper focuses on the role of cell topology in the magnetic properties of a periodic cellular solid. Besides the square cell, additional four cell topologies are investigated for the first time in this paper. For each of them, we present closed-form expressions describing the role of cell topology in the response of a sandwich cylinder subjected to a non-uniform eigenstrain which can assume any arbitrary form. Reference [37], on the other hand, studies the effect of relative density and property gradients for cellular layers with square cells. In addition, this work studies the effect of bonding imperfections on the multiphysics responses of lightweight sandwich cylinders, as opposed to our previous work which assumes perfect bonding among layers. Furthermore, here we present expressions for the effective magnetic permeability that are obtained numerically via standard mechanics and theoretically with closed form bounds. We also conduct a finite element analysis to validate the theoretical predictions. In reference [37], the relative density is assumed to vary linearly and only the Voigt model is used for the calculation; the results presented in this paper show how crude those approximations are, with inaccuracy up to 75% for the effective magnetic permeability.

The paper is organized as follows. Selected planar topologies for the unit cell are examined in Section 2 and their effective magnetoelastic properties are obtained via standard mechanics homogenization. In Sections 3 and 4, a dynamic eigenstrain excitation in the radial direction is expressed as a polynomial with an arbitrary order, which can be applied to any layer of a sandwich cylinder. The results are verified with finite element results and those found in the literature (Sections 5.1 and 5.2). The last part of the paper studies the influence of bonding imperfection, eigenstrain distribution, external magnetic field, cell topology, and relative density, besides mapping these factors in charts that illustrate the time-harmonic responses of alternative sandwich layouts.

## 2. Effective magnetoelastic properties of periodic cellular solids

Homogenization theory is commonly used to determine the effective properties of cellular solids and other periodic materials [54–58]. Many methods have been proposed, including micropolar theory, standard mechanics, asymptotic homogenization, and micromechanical models [36,59], and effectively used to predict the properties of a cellular domain from of a limited portion of it, namely the Representative Volume Element. One main advantage of homogenization methods is the reduced computation cost as compared to a fully detailed analysis, where each cell element would be individually modeled. In this section, we present the effective magnetoelastic properties obtained by standard mechanics (numerical homogenization) and micromechanical closed-form expressions.

For the cellular core of the cylinder, we examine five alternative planar topologies of the unit cell (Figs. 1 and 2): square, mixed (triangular) A, and mixed (triangular) B with cubic symmetry, and Kagome and triangular with isotropic properties. Their material properties are conveniently expressed as a function of their relative density $\rho_r$:

$$\rho_r = \frac{\bar{\rho}}{\rho_s} \tag{1}$$

where $\bar{\rho}$ and $\rho_s$ are, respectively, the effective density of the unit cell of cellular solids and the density of the constituent solid material.

Standard mechanics homogenization with periodic boundary conditions applied to each unit cell (Figs. 1 and 2), along with the classical elasticity theory are here used to obtain the effective magnetoelastic properties numerically. In particular, the effective stiffness and magnetic permeability tensors of an electrically conductive representative volume element (RVE) are here expressed as:

$$\bar{C}_{ijkl} = \frac{1}{V_{RVE}} \int C_{ijmn} M^{C}_{mnkl} dV_{\mathrm{RVE}}, \quad \bar{\mu}_{ij} = \frac{1}{V_{\mathrm{RVE}}} \int \mu_{ik} M^{\mu}_{kj} dV_{\mathrm{RVE}} \tag{2}$$

where $C_{ijkl}$ and $\mu_{ij}$ $(i,j,k,l,m,n = 1,2,3)$ are stiffness and magnetic permeability tensors, $V_{\mathrm{RVE}}$ represents the RVE volume (for a planar RVE, $V_{\mathrm{RVE}}$ is replaced by the area $A_{\mathrm{RVE}}$). Local structural $(M^{C}_{ijkl})$ and local magnetic $(M^{\mu}_{ij})$ tensors are defined as:

$$\varepsilon_{ij} = M^{C}_{ijkl} \bar{\varepsilon}_{kl}, \quad \varphi_{,i} = M^{\mu}_{ij} \bar{\varphi}_{,j} \tag{3}$$

where $\varepsilon_{ij}$ and $\varphi$ represent strain tensor and magnetic potential, respectively [25]. The overbar in Eqs. (1)–(3) stands for the effective properties $(\bar{\rho}, \bar{C}_{ijkl}, \bar{\mu}_{ij})$ or average fields $(\bar{\varepsilon}_{ij}, \bar{\varphi})$. For planar lattices,